Generalized W States and Nonlocal Magic

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Abstract

The complexity of quantum simulations does not arise from entanglement alone. The key aspect of the complexity of the quantum state is shown to be related to non-stabilizerness or magic [1]. The Gottesman-Knill theorem [2] shows that even some highly entangled states can be simulated efficiently. Therefore, magic is a resource and represents the amount of non-Clifford operations (e.g. T-gates) needed to prepare a quantum state.
We demonstrate, using Stabilizer Rényi Entropy [3], that degenerate quantum many-body grounds states with nonzero lattice momentum admit an increment of magic compared to a state with zero momentum [4]. We quantify this increment analytically and show how finite momentum does not only increase the long-range entanglement [5] but also leads to a change in magic. Additionally, we provide a connection between the W state and its generalizations, frequently discussed in the quantum information community, and ground states of frustrated spin chains.

Model and Physics – Frustrated anisotropic XYZ chain

- We consider a spin chain subjected to non-extensive geometrical frustration

$$H=\sum_{j=1}^{L}\left[J_x\sigma_j^x\sigma_{j+1}^x+J_y\sigma_j^y\sigma_{j+1}^y+J_z\sigma_j^z\sigma_{j+1}^z\right]-h\sum_{j=1}^{L}\sigma_j^z. \qquad (1)$$

  - assume PBC $\sigma_{L+1}^\alpha=\sigma_1^\alpha$ with and odd number of spins $L=2M+1$ ($M\in\mathbb{Z}$), with antiferromagnetic coupling $\to$ **Frustrated Boundary Conditions** (FBC) [6]
  - There exists a threshold value $|h|<h^*$ for which the ground-state manifold is at least two-fold degenerate and spanned by states with finite and opposite sign momentum (**chiral region**) [7]
  - Such a manifold is completely described in terms of the eigenstates of the momentum operator P which is the generator of the translation operator T defined, i.e. $T:|\Psi\rangle\to e^{iP}|\Psi\rangle$, whose action shifts all the spins by one site in the lattice and $e^{iP}\neq 1$
  - Ground-state chirality can be characterize by the non-zero expectation value of

$$\langle\Psi|\tau|\Psi\rangle=\langle\Psi|\vec{\sigma}_{i-1}(\vec{\sigma}_i\times\vec{\sigma}_{i+1})|\Psi\rangle \qquad (2)$$

- Simpler examples of frustrated models
  - frustrated Transverse Field Ising model (TFIM) $J_y=J_z=0$ and $J_x=J\to$ **zero momentum GS**
  - frustrated XY chain $J_z=0$ and $J_x=\frac{1+\gamma}{2}$ and $J_y=\frac{1-\gamma}{2}\to$ **finite momentum GS**

Evaluation of magic - Stabilizer Rényi Entropy - SRE

To quantify the amount of non-stabilizerness for a generic state defined on a one-dimensional system made of L qubits/spins, it is possible to use the Stabilizer Rényi-2 Entropy (SRE) [3] that is defined as

$$\mathcal{M}_2(|\psi\rangle)=-\log_2\left(\frac{1}{2^N}\sum_{P}\langle\psi|\mathcal{P}|\psi\rangle^4\right), \qquad (3)$$

where the sum of the r.h.s. runs over all possible Pauli strings $\mathcal{P}=\bigotimes_{j=1}^{L}P_j$ for $P_j\in\{\sigma_j^0,\sigma_j^x,\sigma_j^y,\sigma_j^z\}$ where σ_j^0 stands for the identity operator on the j-th qubit.

Results 1 – Exact results for SRE in finite systems

- **Locality**: SRE well approximated from local quantities such as local magnetization along the z-direction.
  - In case of frustration the local magnetization reads

$$\langle\sigma_j^z\rangle^{(f)}=\langle\sigma_j^z\rangle^{(u)}+\frac{2}{L},$$

  implying that there is a correction (replacing $\langle\sigma_j^z\rangle^{(u)}=m_z$)

$$\mathcal{M}_2(0,L)\simeq L\log_2\left(\frac{1+m_z^2}{1+m_z^4}\right)+4m_z\left(\frac{1}{1+m_z^2}-\frac{2m_z^2}{1+m_z^4}\right).$$

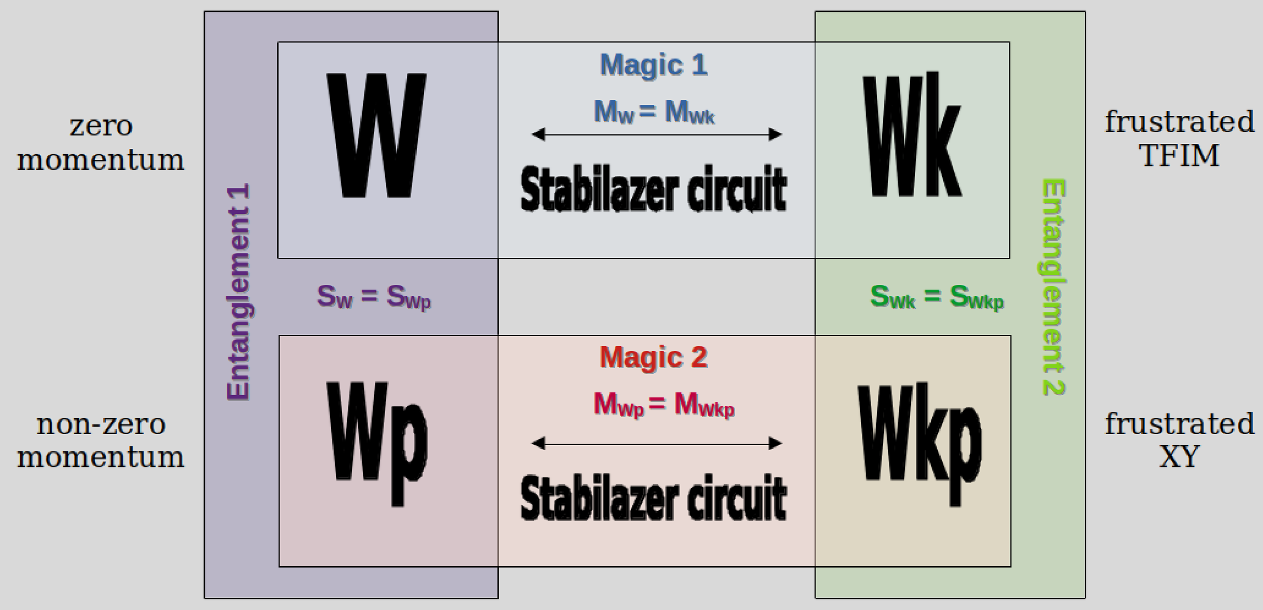
  - **Similar to the magic at QPT!**
- For the zero momentum state we obtain [4]

$$\mathcal{M}_2(0,L)=3\log_2(L)-\log_2(7L-6). \qquad (4)$$

- For finite momentum we obtain (to be published)

$$\mathcal{M}_2(p,L)=-\log_2\left(-\frac{11-12L+\frac{\sin((2-4L)p)}{\sin(2p)}}{2L^3}\right). \qquad (5)$$

Results 2 – From quantum information to condensed-matter physics



- the W state

$$|\mathrm{W}\rangle=\frac{1}{\sqrt{L}}(|100..0\rangle+|010..0\rangle+...+|000...1\rangle) \qquad (6)$$

- the W_k (kink W) state - single domain embedded into Néel states!

$$|W_k\rangle=\frac{1}{\sqrt{2L}}(|0010101...\rangle+|10010101..\rangle+...|1010101...00\rangle+|110101010..\rangle+|01101010...\rangle+...+|0101010...11\rangle) \qquad (7)$$

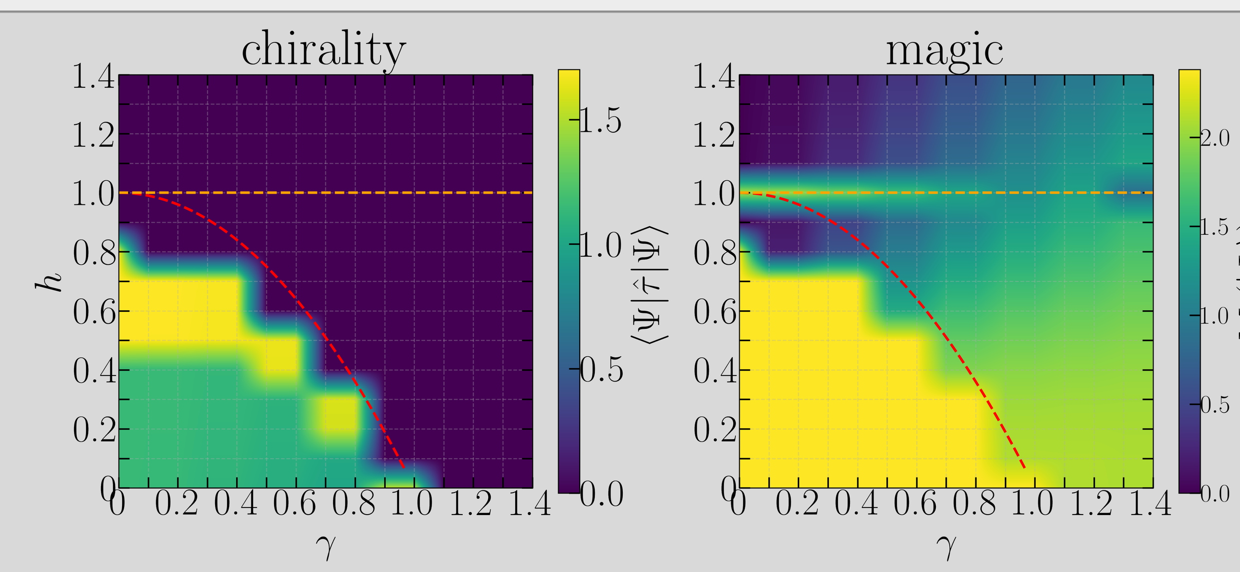
- generalized W state to finite momentum

$$|W_p\rangle=\frac{1}{\sqrt{N}}(e^{ip}|100..0\rangle+e^{2ip}|010..0\rangle+...+e^{Lip}|000...1\rangle) \qquad (8)$$

- generalized W_k state to finite momentum

$$|W_{kp}\rangle=\frac{1}{\sqrt{2L}}(e^{ip}|0010101...\rangle+e^{2ip}|10010101..\rangle+...+e^{-ip}|110101010..\rangle+e^{-2ip}|01101010...\rangle+...) \qquad (9)$$

Results 3 – SRE as 'order' parameter in the thermodynamic limit



$$\Delta\mathcal{M}_2(L)\equiv\mathcal{M}_2(p,L)-\mathcal{M}_2(0,L)=\log_2\left(\frac{7L-6}{6L-6}\right) \qquad (10)$$

In the thermodynamic limit, we obtain

$$\lim_{L\to\infty}\Delta\mathcal{M}_2(L)=\log_2\left(\frac{7}{6}\right) \qquad (11)$$

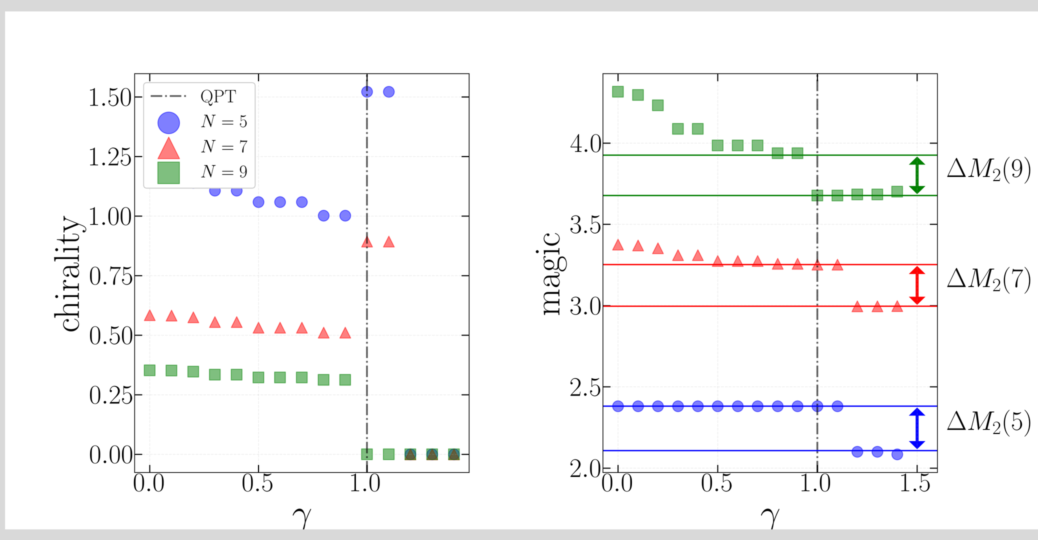


Figure: Comparison between chirality and SRE for finite system sizes in the classical limit of the frustrated XY model ($h\to 0^+$). The chirality vanishes in the thermodynamic limit while SRE stays finite (see Eq. 11).

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