Strings, Holography & Chaos group seminar

Quantum chaos, non-stabilizerness and the Sachdev-Ye-Kitaev model

Jovan Odavić - University of Naples, Federico II - Italy

Enabled by PNRR MUR project PE0000023-NQSTI and LEONARDO supercomputer, hosted by CINECA (Italy) under the project ID: PQC - HP10CQQ3SR.

Checklist

- Introduction to quantum chaos and non-stabilizerness (“magic”)
    - Sachdev-Ye-Kitaev model
        - Spectral Form Factor and random walk

arXiv > quant-ph > arXiv:2502.03093

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Quantum Physics

[Submitted on 5 Feb 2025 (v1), last revised 23 Feb 2025 (this version, v2)]

Stabilizer Entropy and entanglement complexity in the Sachdev–Ye–Kitaev model

Barbara Jasser, Jovan Odavic, Alioscia Hamma

The Sachdev–Ye–Kitaev (SYK) model is of paramount importance for the understanding of both strange metals and a microscopic theory of two–dimensional gravity. We study the interplay between Stabilizer Rényi Entropy (SRE) and entanglement entropy in both the ground state and highly excited states of the SYK4+SYK2 model interpolating the highly chaotic four–body interactions model with the integrable two–body interactions one. The interplay between these quantities is assessed also through universal statistics of the entanglement spectrum and its anti–flatness. We find that SYK4 is indeed characterized by a complex pattern of both entanglement and non–stabilizer resources while SYK2 is non–universal and not complex. We discuss the fragility and robustness of these features depending on the interpolation parameter.

arXiv:2502.03093

arXiv > quant-ph > arXiv:2505.05199

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Quantum Physics

[Submitted on 8 May 2025 (v1), last revised 21 May 2025 (this version, v2)]

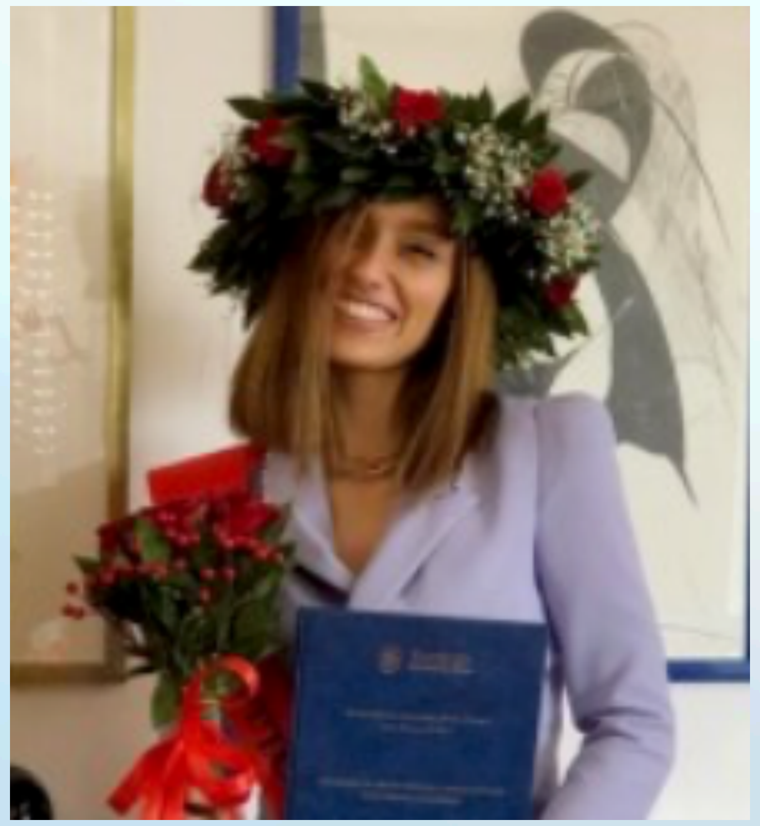
Integrability and Chaos via fractal analysis of Spectral Form Factors: Gaussian approximations and exact results

Lorenzo Campos Venuti, Jovan Odavić, Alioscia Hamma

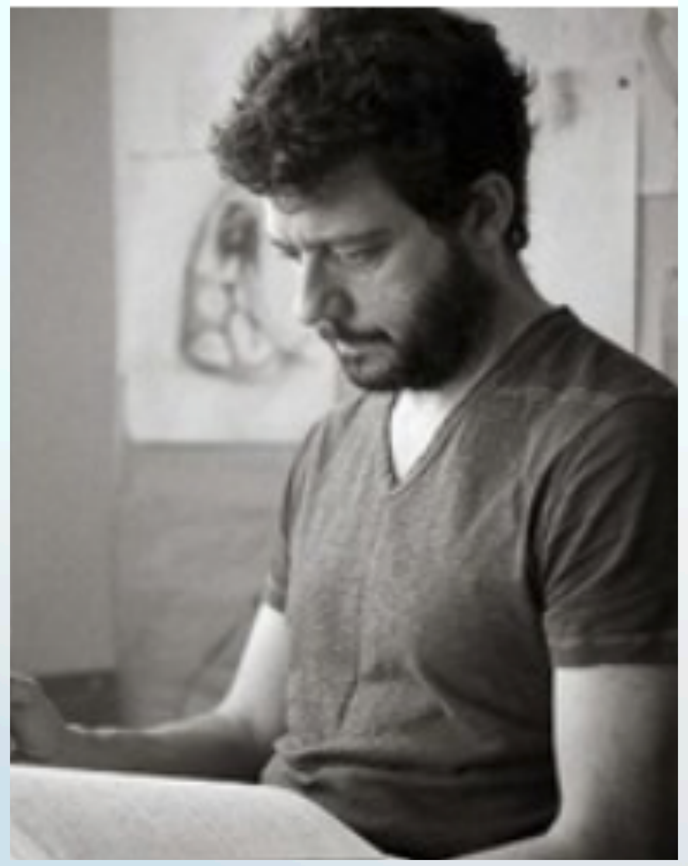
We establish the mathematical equivalence between the spectral form factor, a quantity used to identify the onset of quantum chaos and scrambling in quantum many–body systems, and the classical problem of statistical characterization of planar random walks. We thus associate to any quantum Hamiltonian a random process on the plane. We set down rigorously the conditions under which such random process becomes a Wiener process in the thermodynamic limit and the associated distribution of the distance from the origin becomes Gaussian. This leads to the well known Gaussian behavior of the spectral form factor for quantum chaotic (non–integrable) models, which we show to be violated at low temperature. For systems with quasi–free spectrum (integrable), instead, the distribution of the SFF is Log–Normal. We compute all the moments of the spectral form factor exactly without resorting to the Gaussian approximation. Assuming degeneracies in the quantum chaotic spectrum we solve the classical problem of random walker taking steps of unequal lengths. Furthermore, we demonstrate that the Hausdorff dimension of the frontier of the random walk, defined as the boundary of the unbounded component of the complement, approaches 1 for the integrable Brownian motion, while the non–integrable walk approaches that obtained by the Schramm–Loewner Evolution (SLE) with the fractal dimension 4/3. Additionally, we numerically show that Bethe Ansatz walkers fall into a category similar to the non–integrable walkers.

arXiv:2505.05199

Lorenzo Campos Venuti



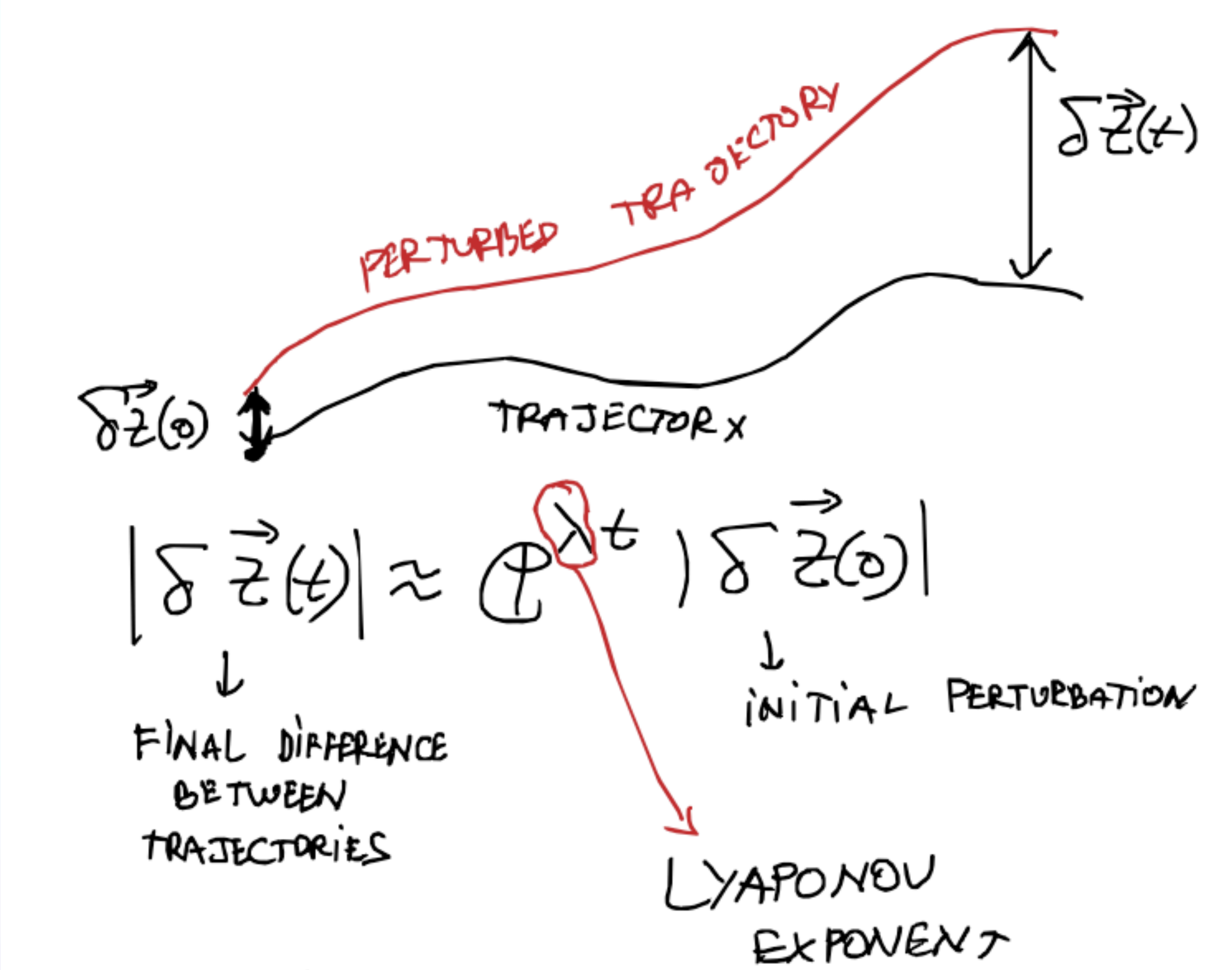
Barbara Jasser



Alioscia Hamma

(Quantum) Chaos refresher

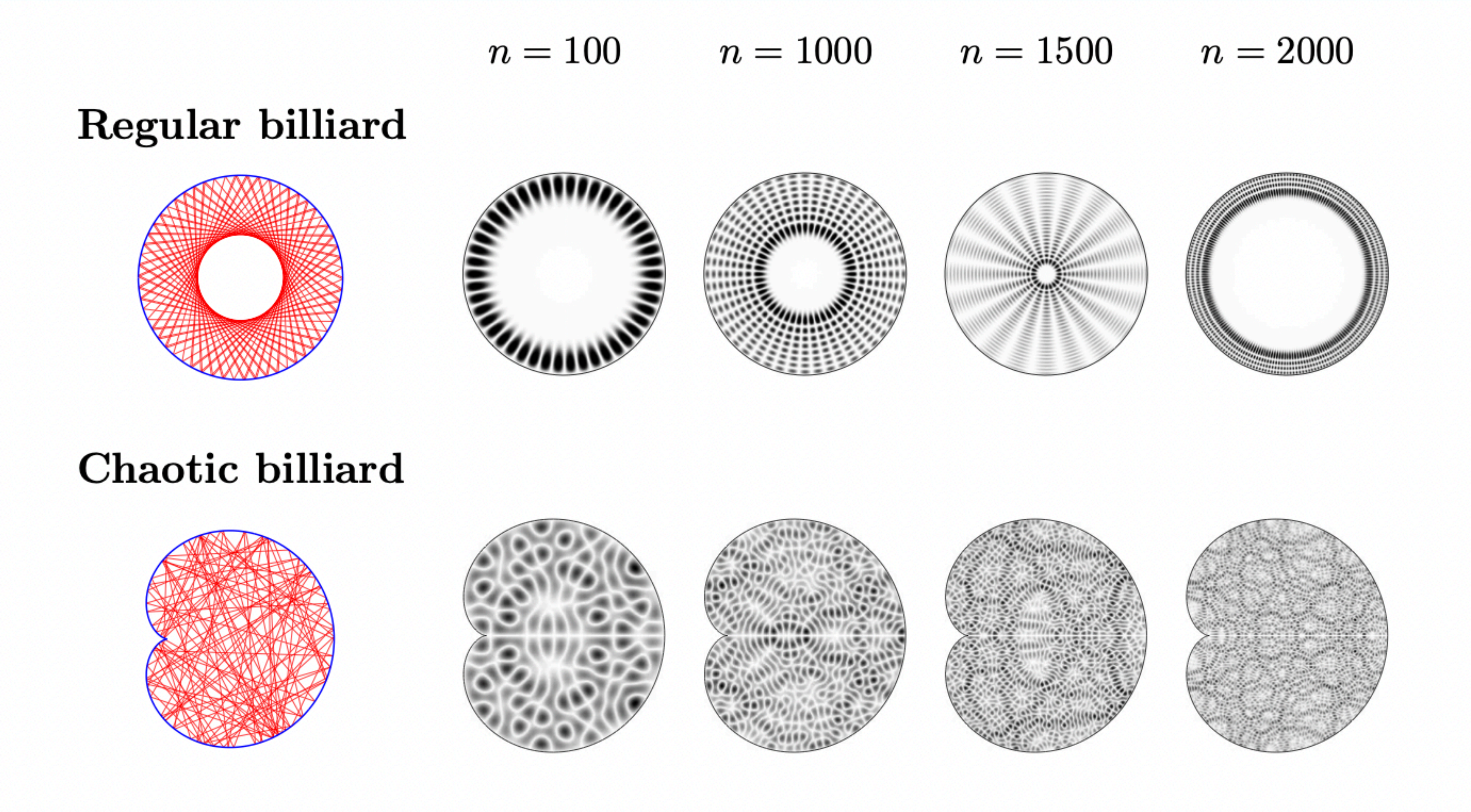
- In <u>classical systems</u> **chaos** arises from **non-linearity** dynamics and **strong interactions** between degrees of freedom that cause particle trajectories to diverge, resulting in ergodicity within the accessible phase space.

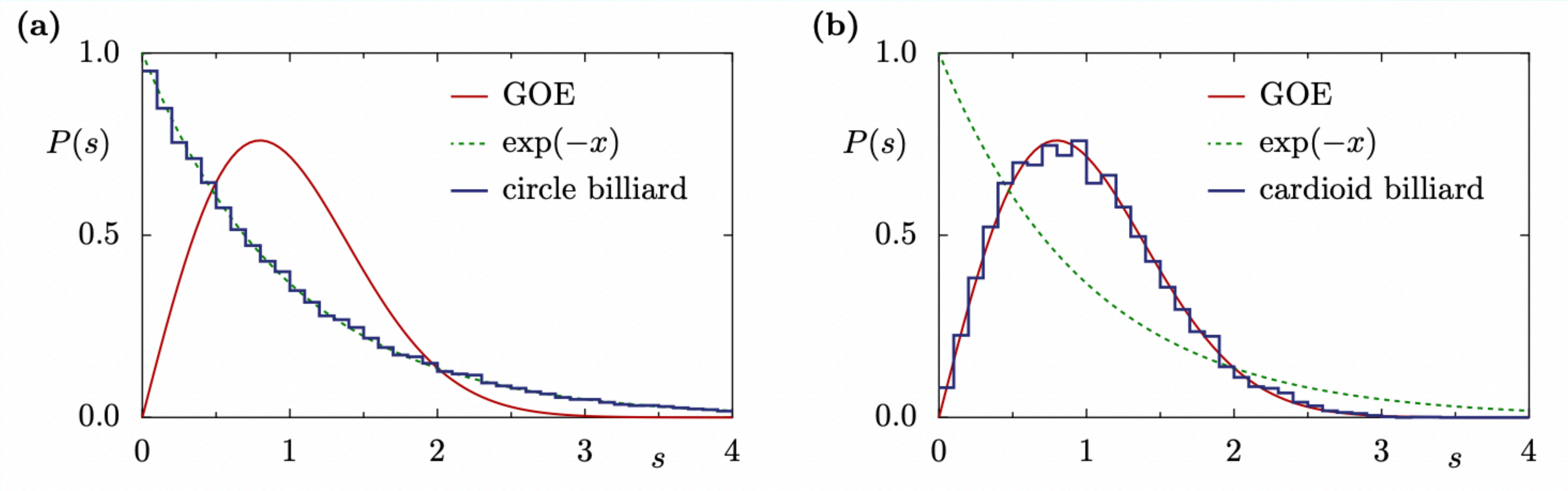


- In <u>quantum systems</u> its much harder to define due to apparent reversibility of quantum evolution in closed systems

Single particle quantum chaos

- Classical/Quantum billiard example —> Lyapunov exponents and integrability





Classical integrable dynamics quantized —> Poisson level statistics
Classical non-integrable (chaotic) dynamics quantized —> Gaussian RMT

- Commonly accepted probe for quantum chaos is spectral statistics to match of that of correlated Gaussian RMT ensembles (universal statistics of gaps) - Bohigas–Giannoni–Schmit (BGS) conjecture
- RMT = linear algebra + probability
- For many-body system remains a numerically verified conjecture

from "Quantum chaos in billiards" - Arnd Backer

Quantum chaos (QC) in many-body systems is crucial to a variety of scientific fields

1. *<u>Statistical physics</u>*: QC induces eigenstate typicality by generating randomness in the structure of energy eigenstates, which forces them to exhibit universal statistical properties matching thermal ensembles - Eigenstate Thermalization Hypothesis (ETH); M. Rigol, V. Dunjko, and M. Olshanii, Nature **452**, 854–858 (2008), Mark Srednicki, Phys. Rev. E **50**, 888 (1994), L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, Advances in Physics **65**, 239–362 (2016).

2. *<u>Black hole physics</u>*: QC is connected to the fast information spreading across the available d.o.f.; Y. Sekino and L. Susskind, Journal of High Energy Physics 2008, 065 (2008), J. Maldacena, S. H. Shenker, and D. Stanford, Journal of High Energy Physics 2016, 106 (2016).

3. *<u>Condensed-matter and many-body physics</u>*: efficient simulation using tensor network methods due to fast growth of entanglement (hallmark of QC); J. Ignacio Cirac, David Pérez-García , Norbert Schuch, and Frank Verstraete, Rev. Mod. Phys. **93**, 045003 (2021)

4. Q*<u>uantum computation:</u>* DiVincenzo's criteria - universal quantum gate set i.e. ability to explore the full Hilbert space ergodically; D. P. DiVincenzo, Fortschritte der Physik **48**, 771–783 (2000), L. Leone, S. F. E. Oliviero, Y. Zhou, and A. Hamma, Quantum **5**, 453 (2021).

... and others

Quantum information perspective on quantum chaos (Clifford+T gates quantum circuits)

- For a particular subclass of quantum circuits, simulating entanglement is **efficiently performed** on a classical computer (Gottesman, D. (1998) 'The Heisenberg Representation of Quantum Computers') also known as stabilizer circuits
- Quantum circuits involving only **Clifford** operations can be simulated efficiently even if the final states are **highly entangled**.
- Entanglement is not the full picture then!
- Not **every** quantum circuit can be done with Cliffords
- Google supremacy experiment: "Quantum supremacy using a programmable superconducting processor", Nature **574** 505-510 (2019) using Clifford circuits

Hadamard gate

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

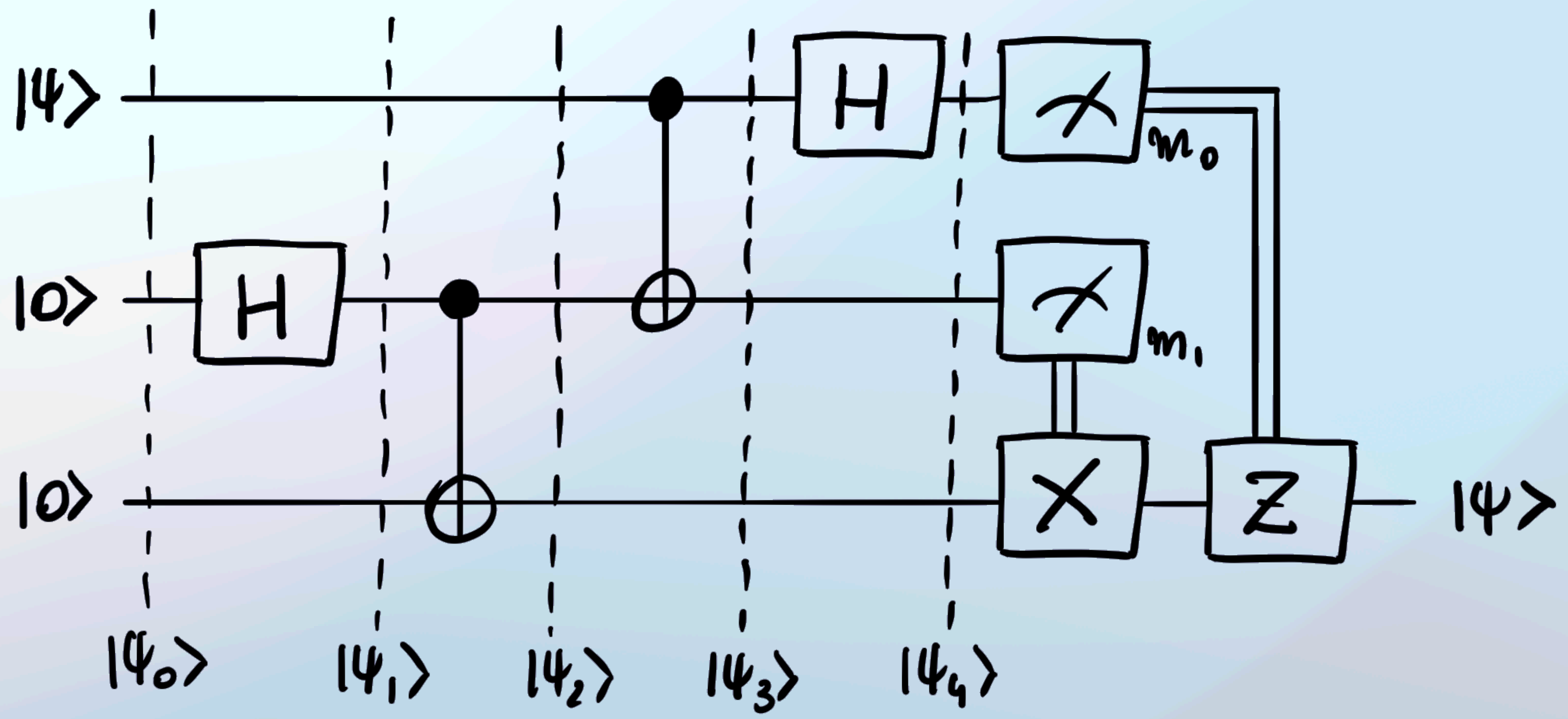
Phase gate

$$S = \begin{bmatrix}1 & 0\\ 0 & e^{-i\pi/2}\end{bmatrix}$$

Control+NOT

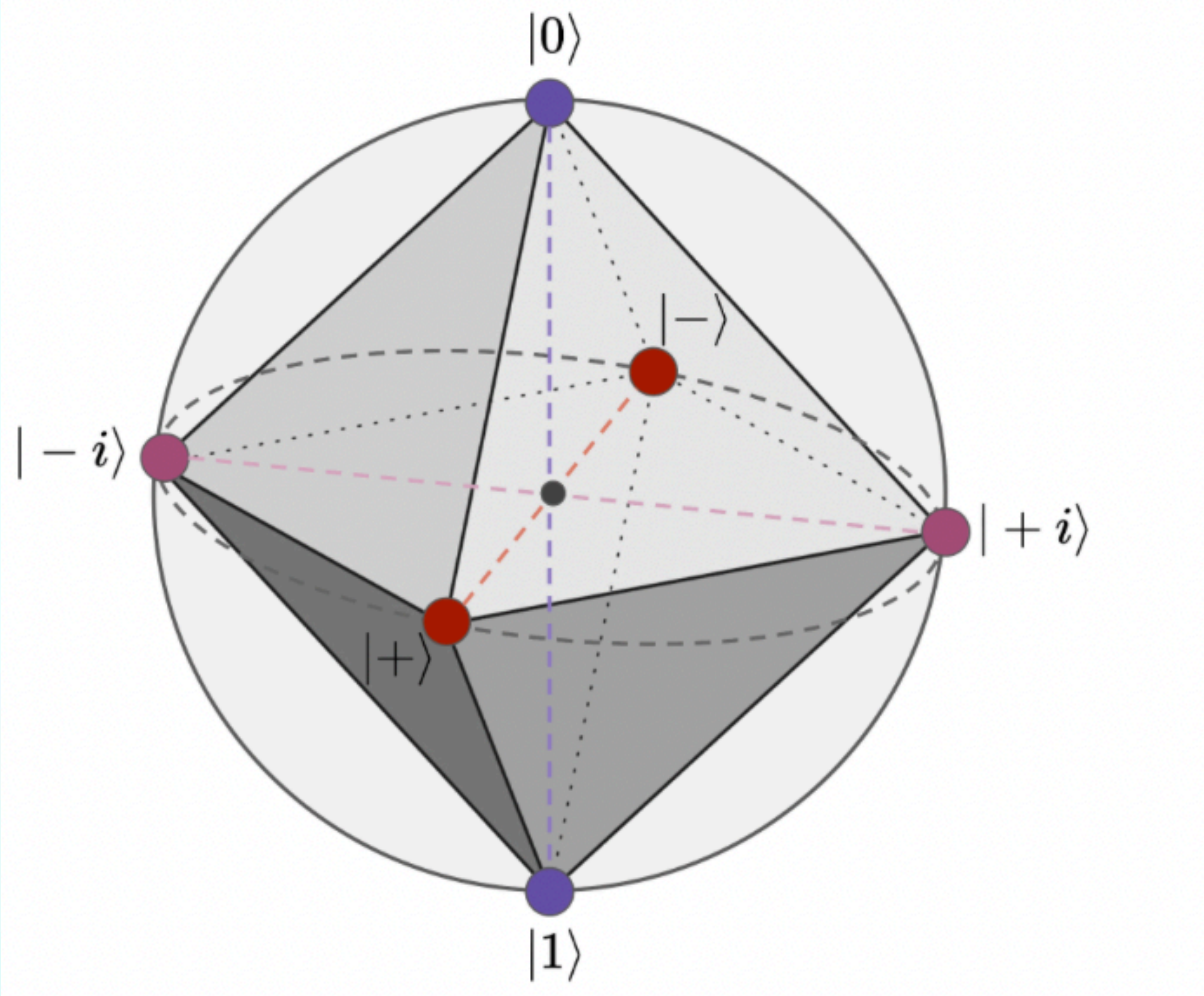
$$\text{CNOT} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$$

quantum state teleportation protocol



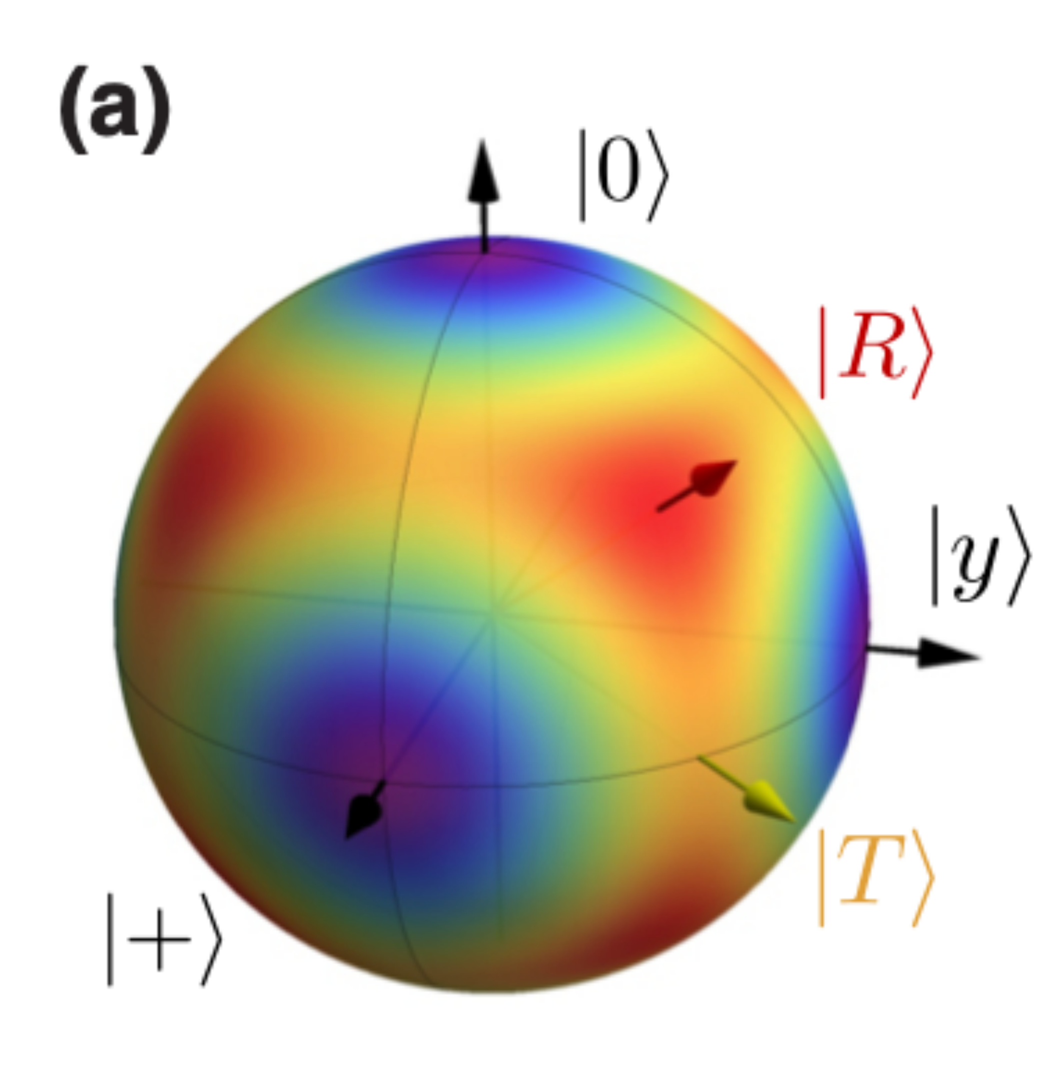
Single qubit example

only exploring small patches of the full Hilbert space —> 6 pure stabilizer state in case of 1 qubit



The octahedron in the Bloch sphere defines the states accessible via single-qubit Clifford gates (https://pennylane.ai/qml/demos/tutorial_clifford_circuit_simulations).

non-clifford gates allows for the exploration of the full Hilbert space



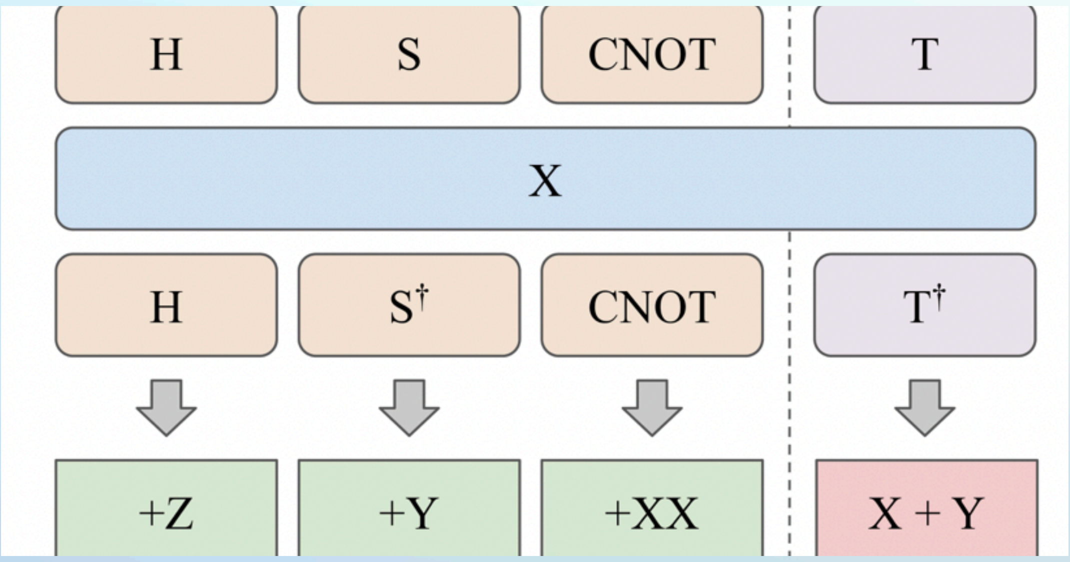
PRX QUANTUM 4, 010301 (2023)

The paradigm of Clifford+T circuits

- Gottesman's method of simulating quantum systems breaks down when **T gates** are added to the mix
- Systems become exponentially more difficult to simulate as more magic is present; Aaronson, S. and Gottesman, D. 'Improved Simulation of Stabilizer Circuits', Physical Review A, **70**(5) (2004)
- Only applying T gates gets you to explore the full Hilbert space (Nielsen and Chuang)

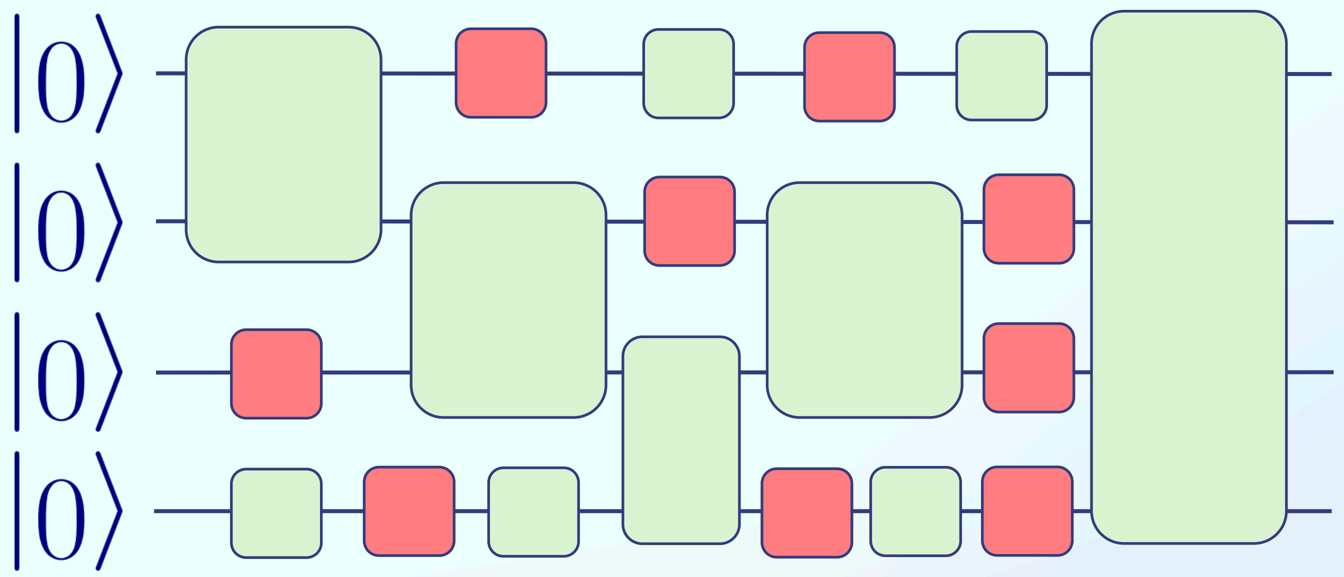
With T-gates definition

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

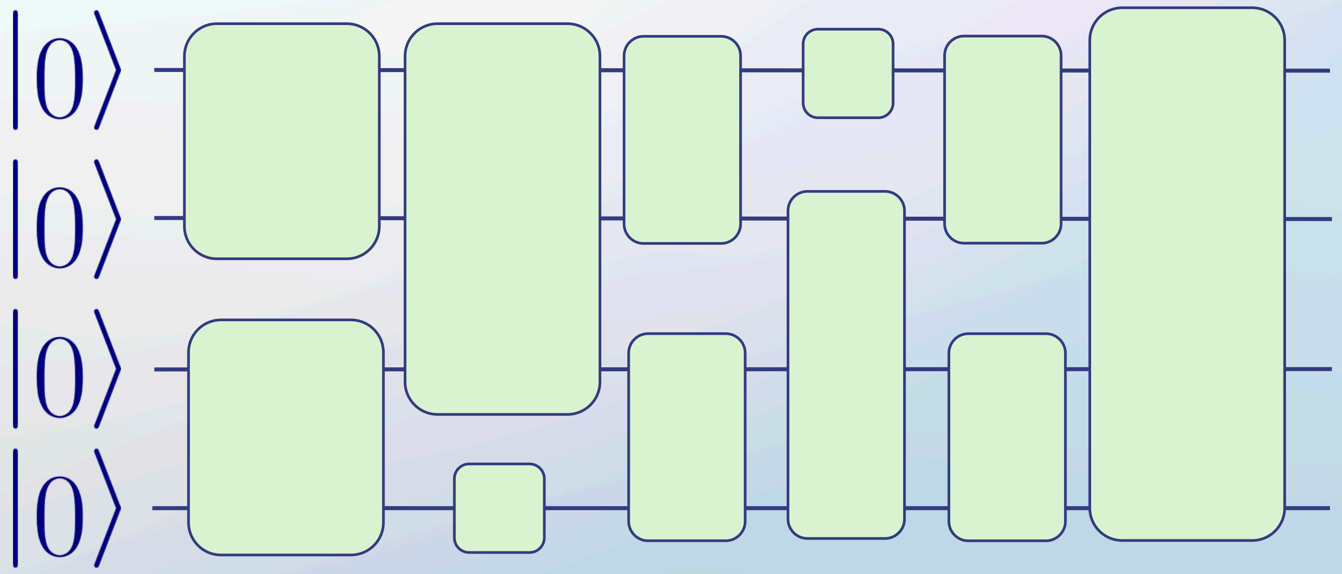


(https://pennylane.ai/qml/demos/tutorial_clifford_circuit_simulations).

resources required for simulation



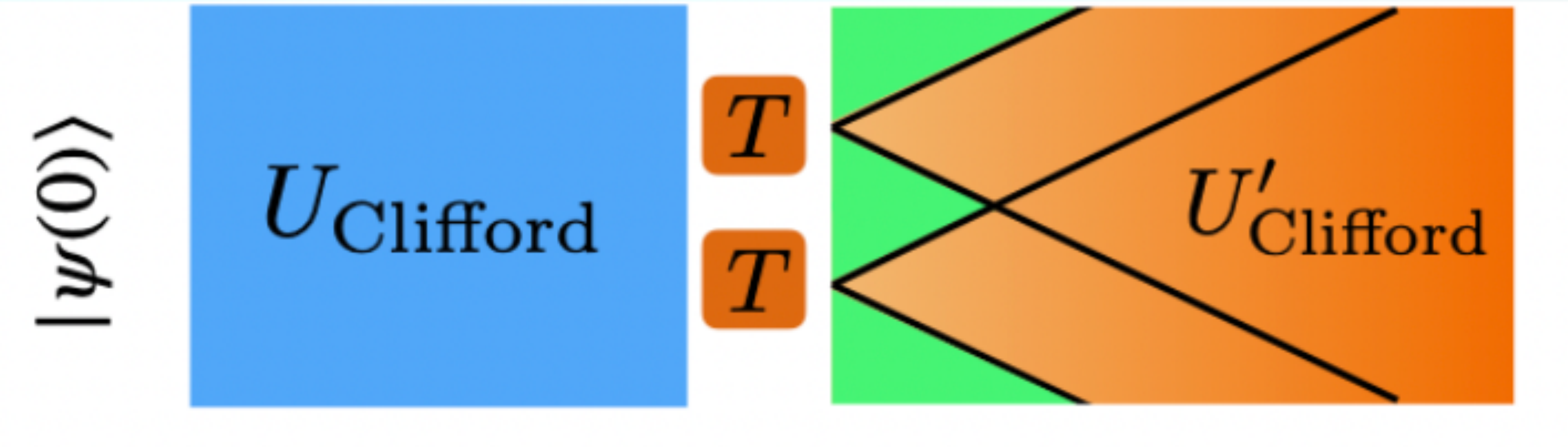
$\exp(n)$ magic



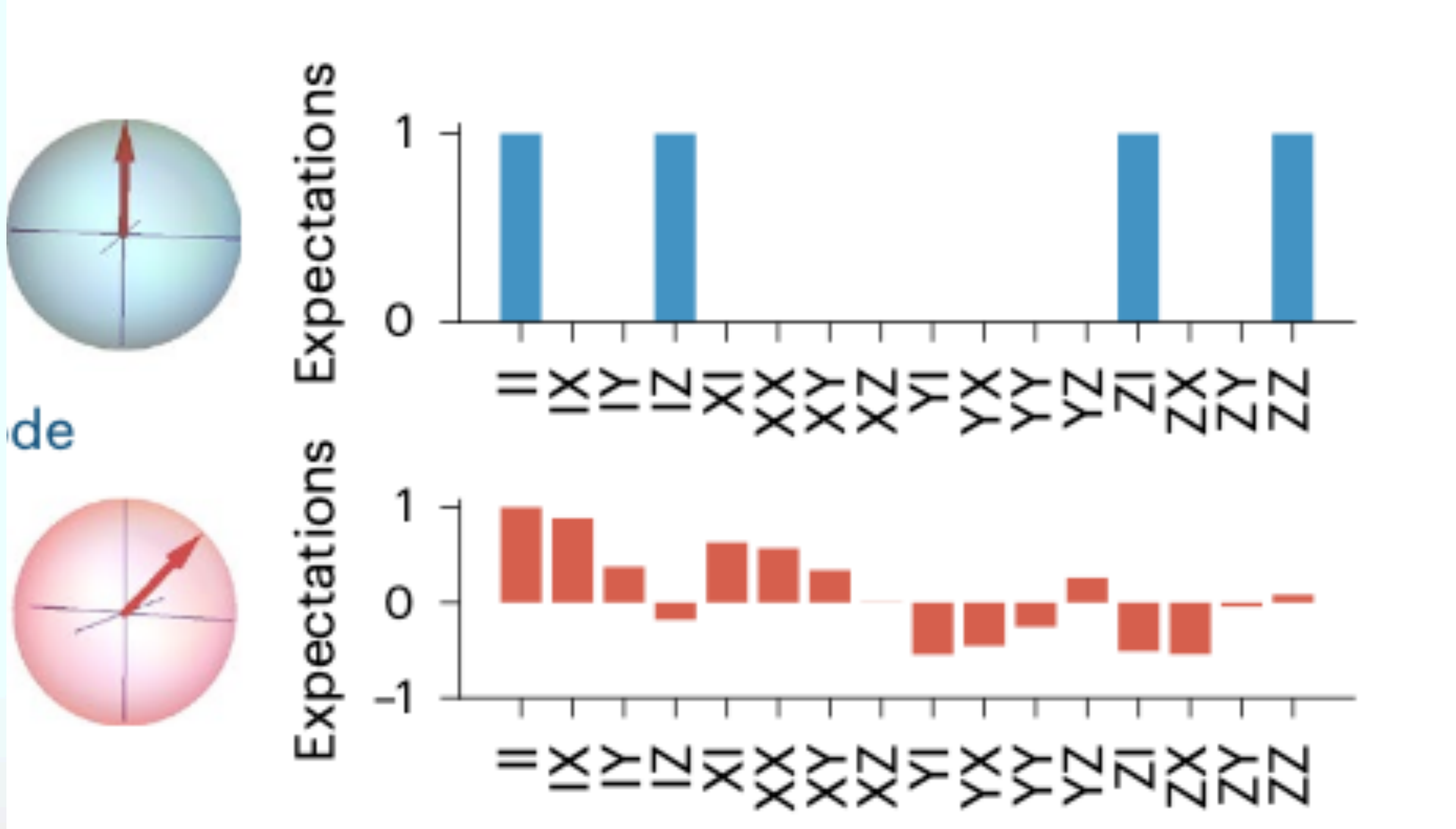
$\text{poly}(n)$ no magic

The effects of T-gate doping

Universality enabling gate set



SciPost Phys. 9, 087 (2020)



Niroula, P., White, C.D., Wang, Q. *et al.* Phase transition in magic with random quantum circuits. *Nat. Phys.* 20, 1786–1792 (2024)

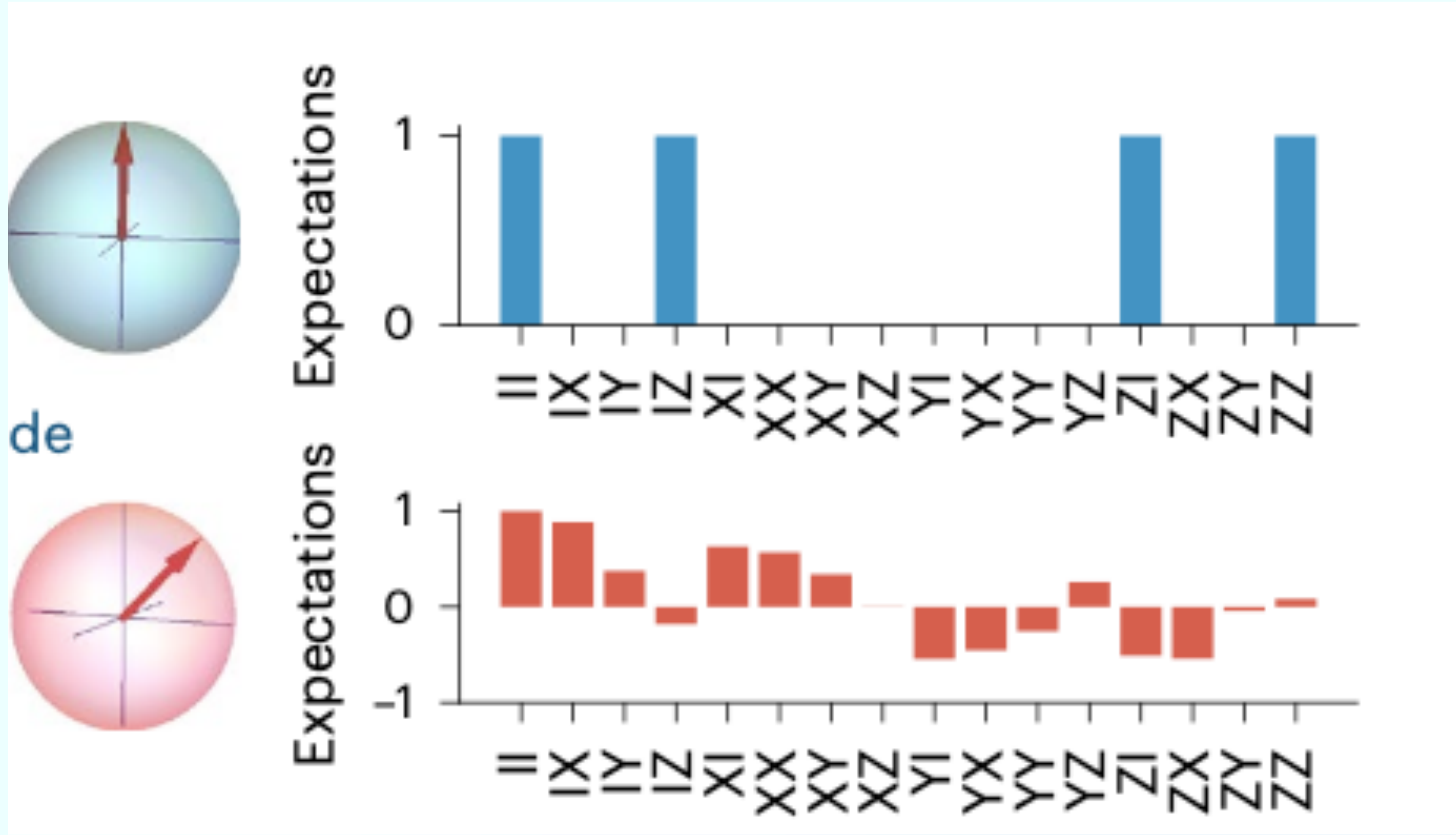
Stabilizer Entropy

L. Leone, S.F.E. Oliviero, A. Hamma PRL **128**, 050402 (2022)

Efficiently measurable quantity of magic in multi-qubit systems

$$M_\alpha(\rho) = \frac{1}{1-\alpha}\log_2\left(\sum_{P\in P_N} d^{-1}|\mathrm{Tr}(\rho P)|^{2\alpha}\right)$$

- Faithfulness : $M_\alpha(\rho) = 0$ iff $\rho \in \mathrm{STAB}$
- Stability : $M_\alpha(C\rho C^\dagger) = M_\alpha(\rho)$
- Additivity : $M_\alpha(\rho \otimes \phi) = M_\alpha(\rho) + M_\alpha(\phi)$



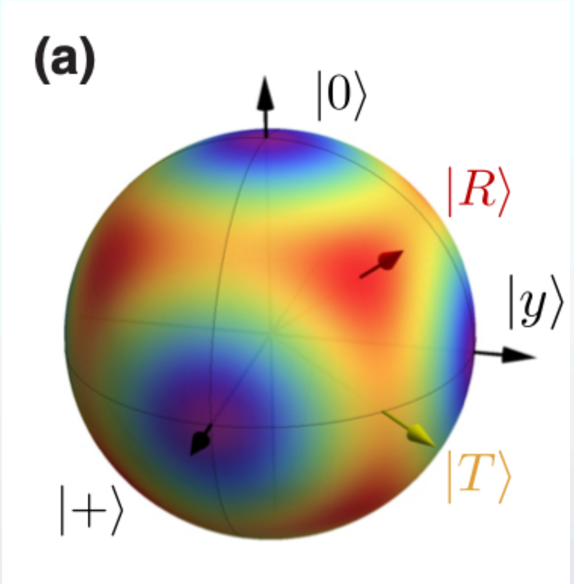
Niroula, P., White, C.D., Wang, Q. *et al.* Phase transition in magic with random quantum circuits. *Nat. Phys.* 20, 1786–1792 (2024)

– Does not require require different minimization procedure compared to quantities scubas as robustness of magic, mana, stabilizer nullity …

- Computable using modern Matrix Product State (MPS) and tensor network algorithms

Checklist status

- Introduction to quantum chaos and non-stabilizerness ("magic") ✓
- Sachdev-Ye-Kitaev model ✗
- Spectral Form Factor and random walk ✗



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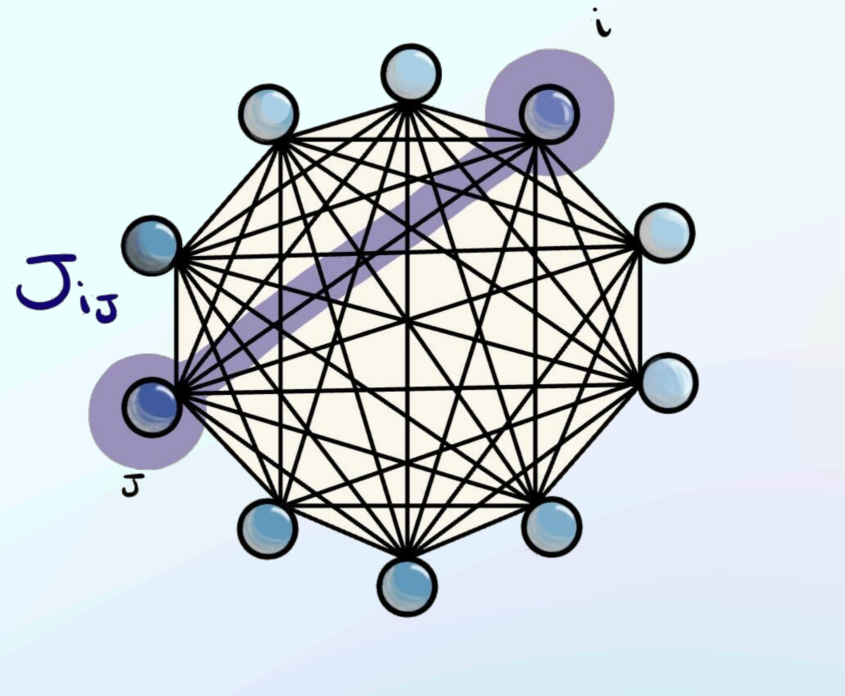
4, 010301 (2023)

Sachdev-Ye-Kitaev model (arXiv:2502.03093)

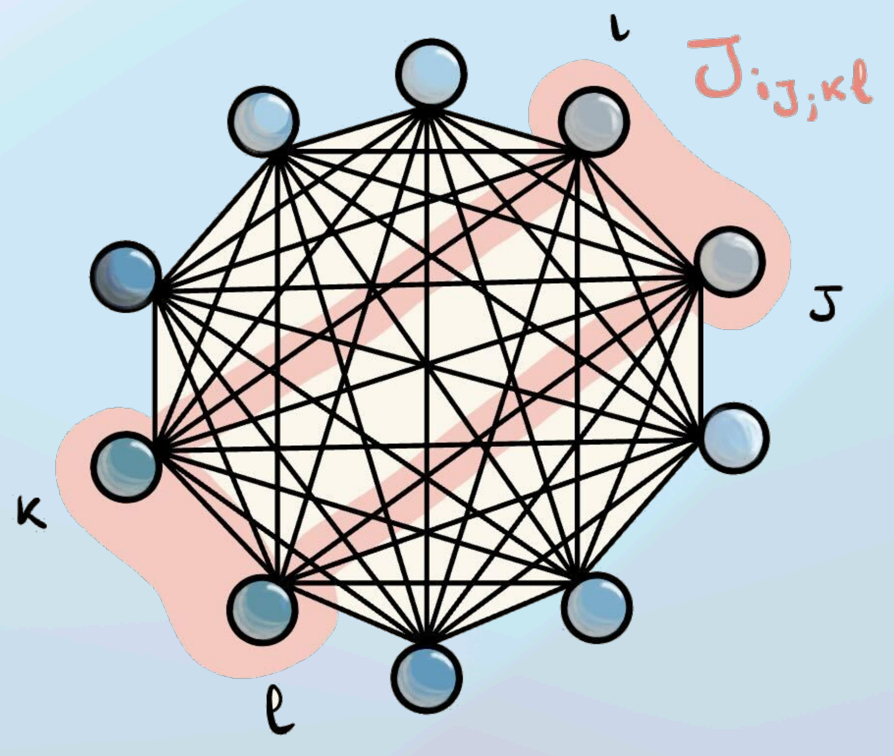
Finite dimensional disordered all-to-all coupled (0+1)D many-body model

$$H^{\mathrm{SYK}-q} = (i)^{q/2} \sum_{1\le i_1<\ldots<i_q\le N} J_{i_1 i_2 \ldots i_q} \chi_{i_1}\chi_{i_2}\cdots\chi_{i_q}$$



Majorana operators following the commutation algebra $\{\chi_i, \chi_j\} = \delta_{i,j}, \quad \text{and} \quad \chi_i^\dagger = \chi_i \qquad i, j = 1, \ldots, N$

The disorder couplings have the mean and variance $\overline{J_{i_1,i_2,\ldots,i_q}} = 0\,, \quad \overline{J^2_{i_1,i_2,\ldots,i_q}} = \frac{(q-1)!J}{N^{q-1}}$



$$P(J_{i_1 i_2 i_3 i_4}) \simeq \exp\left(-\frac{N^3 J^2_{i_1 i_2 i_3 i_4}}{12 J^2}\right), \qquad \forall J_{i_1 i_2 i_3 i_4}\,.$$

1993—>2015

S. Sachdev and J. Ye, Physical Review Letters 70, 3339–3342 (1993)., KITP, Proceedings of the kitp (2015), http://online.kitp.ucsb.edu/online/entangled15/kitaev/, http://online.kitp.ucsb.edu/online/entangled15/kitaev2/; D. Chowdhury, A. Georges, O. Parcollet, and S. Sachdev, Reviews of Modern Physics 94, 035004 (2022).

Interesting facts about the model

Initial intended to describe the behavior of strongly correlated fermions in strange metals.

SYK is connected to models of low dimensional gravity and effective descriptions of black holes close to singularity —> Jackiw-Teitelboim (JT) theory and nearly-AdS2 spaces

- two point correlations, four point correctors, density of states match between the disordered quantum model and the gravity model

- The effective action of the theories are the same, implying by solving the quantum mechanical problem on one side we are able to say something about gravity on the other

- The SYK-4 variant saturates the bound on complexity (fast scrambling) of the out-of-time-ordered correlation (OTOC) functions and provides a maximal quantum Lyapunov exponent

More interesting facts

Exactly Solvable in the large N limit enabled by gaussian disorder in the couplings —>

resummation of Feynman diagram in a precise way

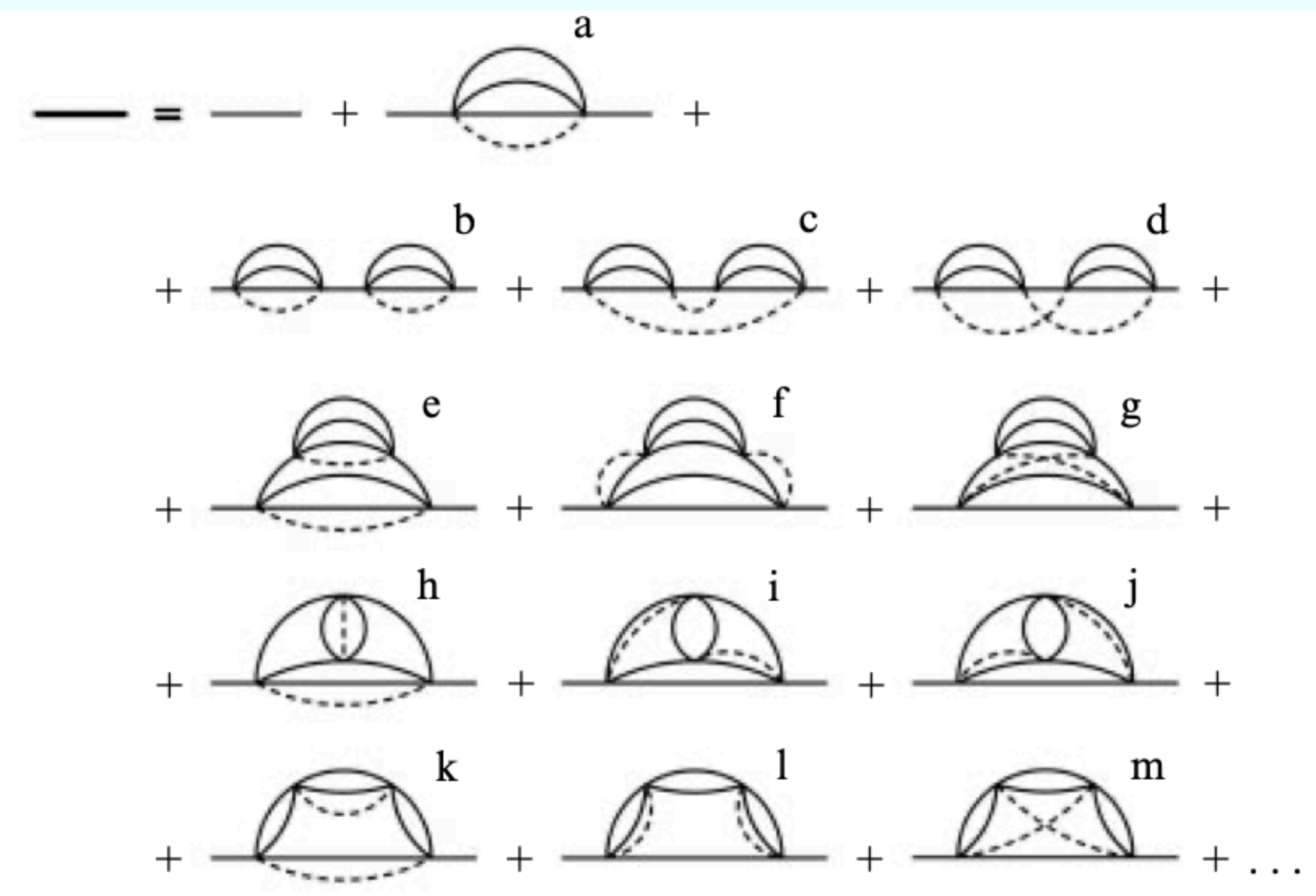


Figure 3. (a) Second-order and (b–m) fourth-order corrections to the propagator. The only diagrams that survive in the limit $N \to \infty$ are (a), (b), and (e).

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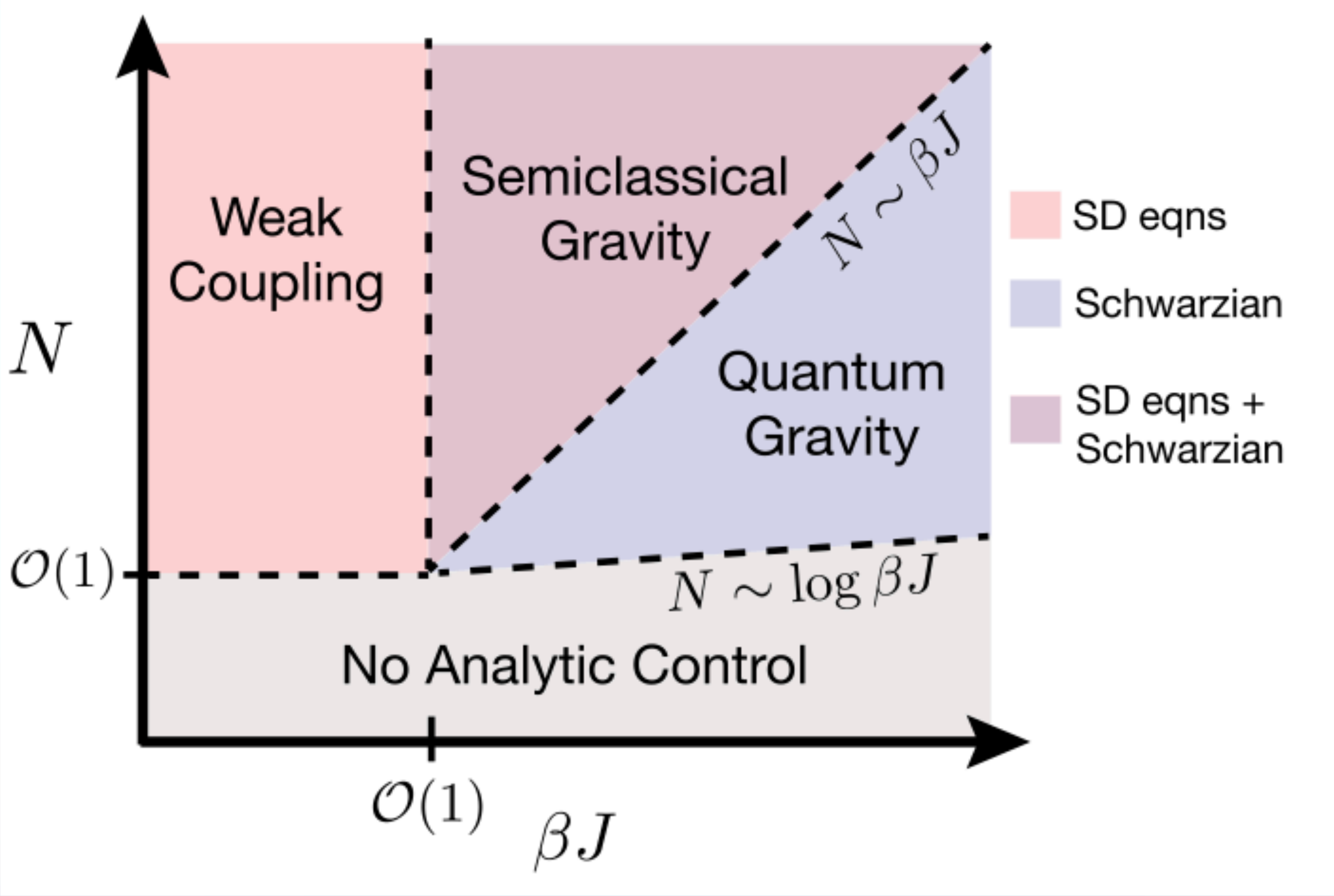


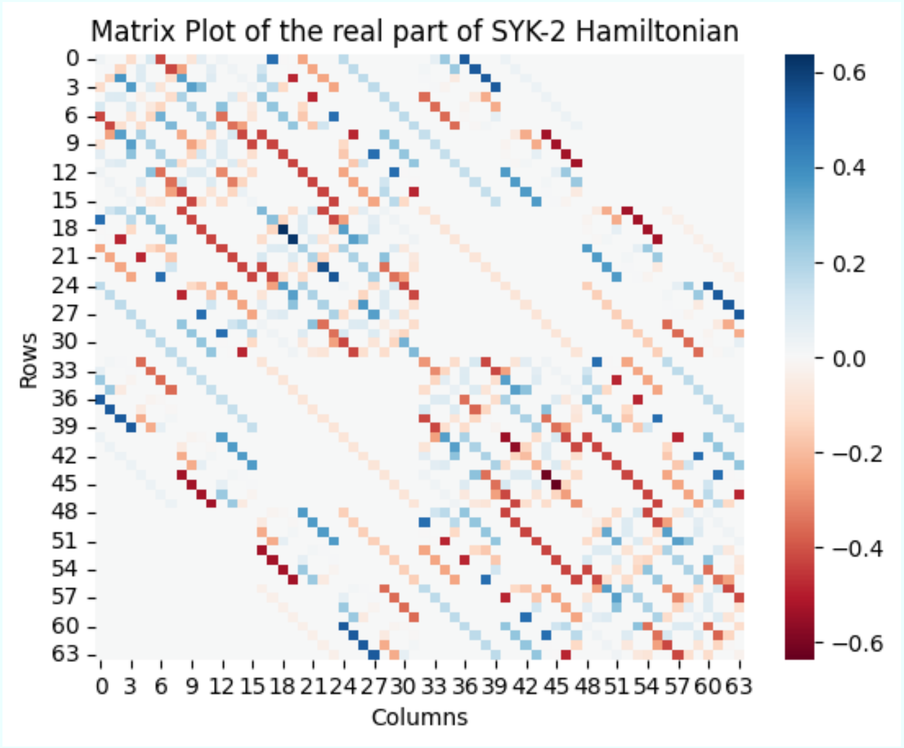
Solvable limits of the SYK-4 model - PRL **126**, 030602 (2021)

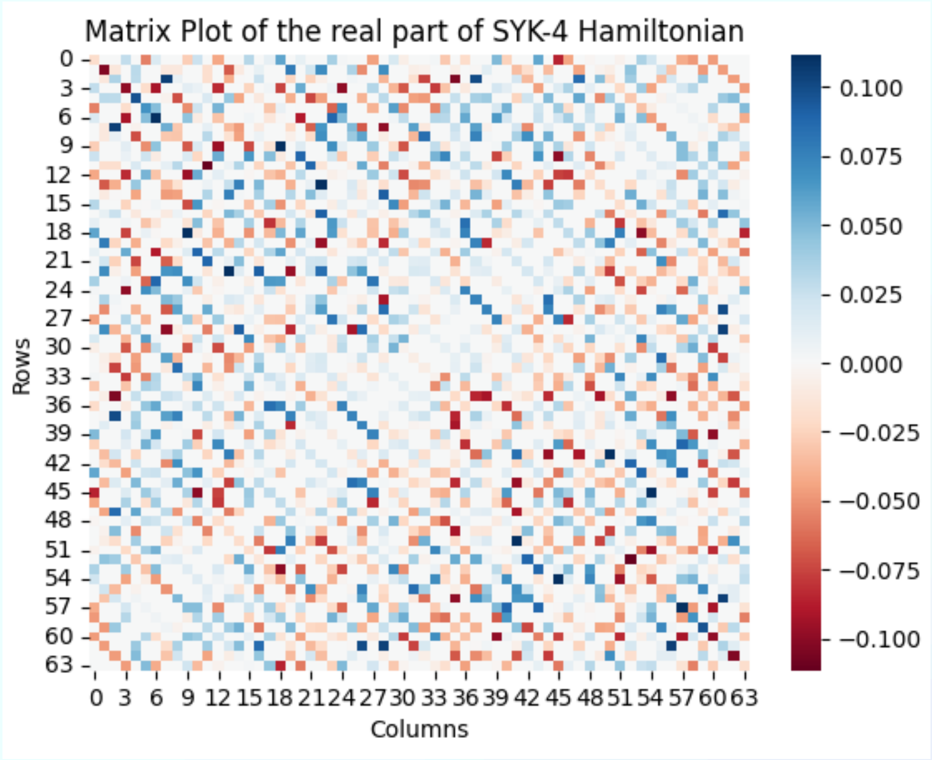
Numerical approach

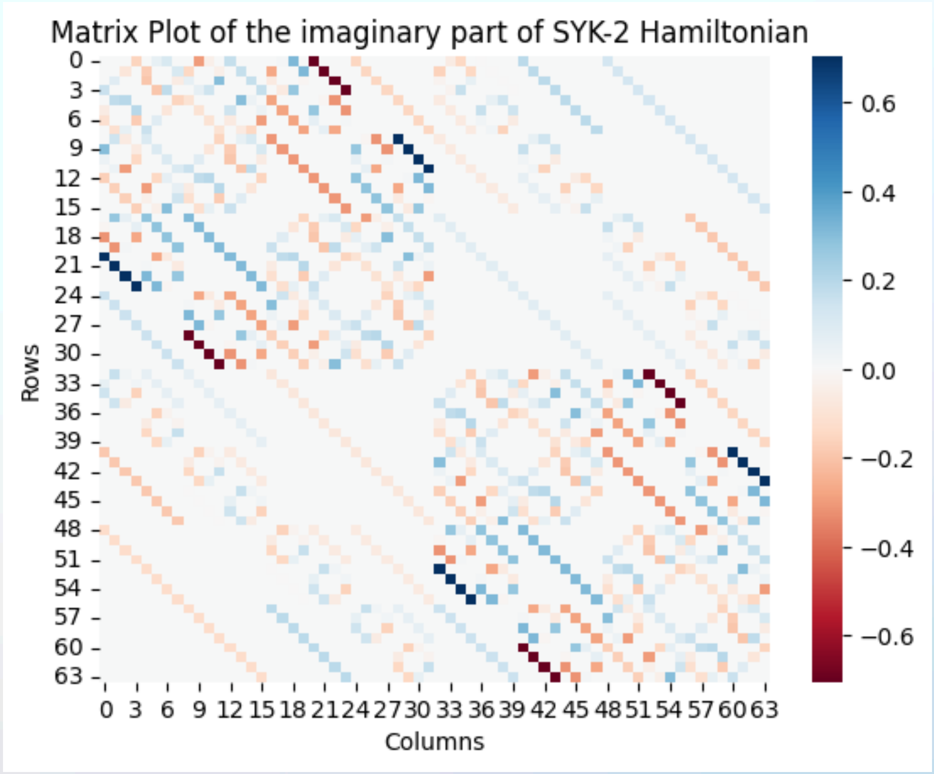
- Jordan-Wigner transformations :

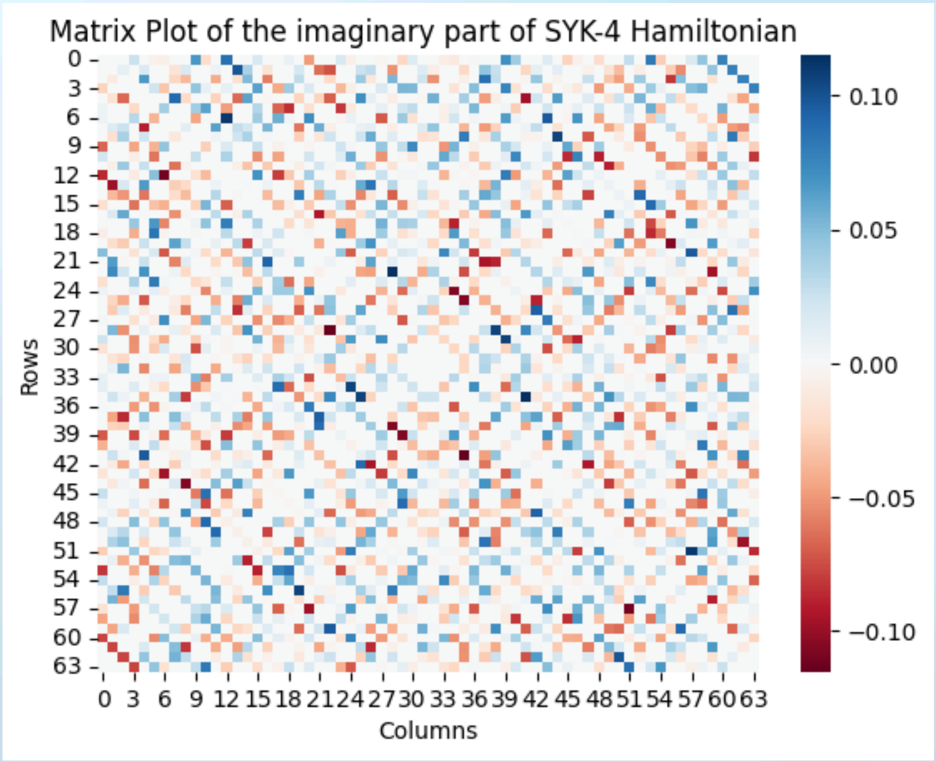
$$\hat{\chi}_{2n} = \hat{c}_n + \hat{c}_n^{\dagger} = \left(\bigotimes_{j=0}^{n-1} \hat{\sigma}_j^z\right) \otimes \hat{\sigma}_n^x \otimes \hat{\mathbb{I}}_{n+1} \otimes \ldots \otimes \hat{\mathbb{I}}_N \quad ,$$

$$\hat{\chi}_{2n+1} = \frac{\hat{c}_n - \hat{c}_n^{\dagger}}{i} = \left(\bigotimes_{j=0}^{n-1} \hat{\sigma}_j^z\right) \otimes \hat{\sigma}_n^y \otimes \hat{\mathbb{I}}_{n+1} \otimes \ldots \otimes \hat{\mathbb{I}}_N$$

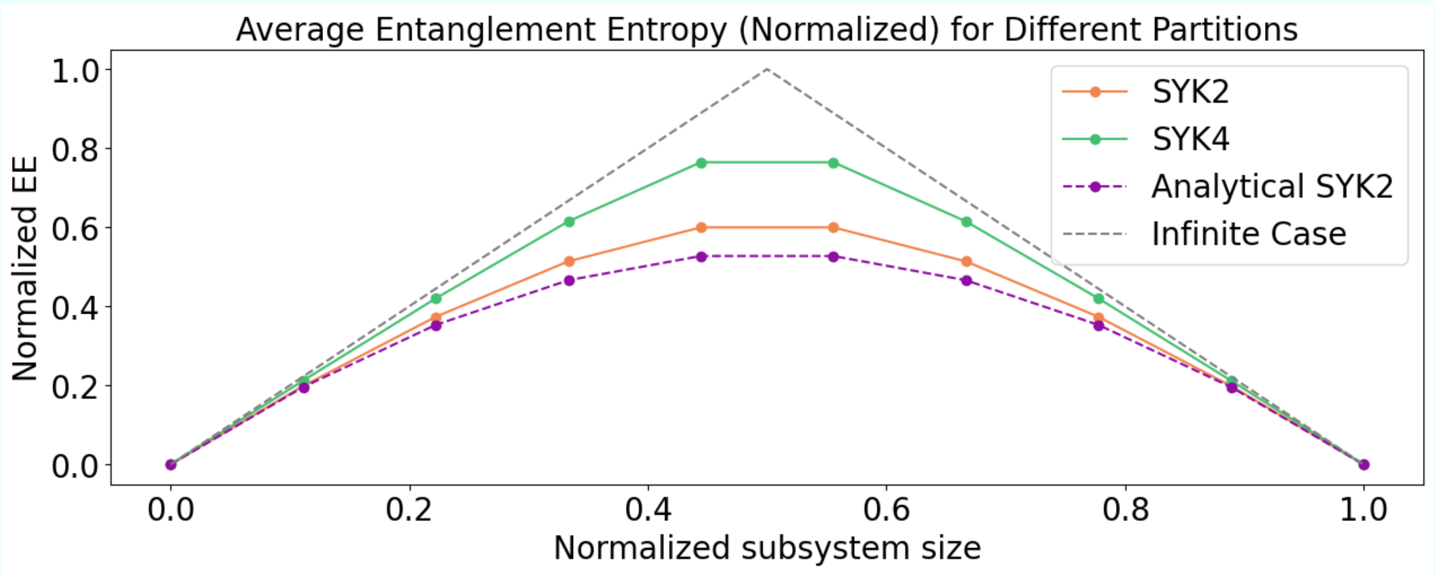








Interpolated SYK



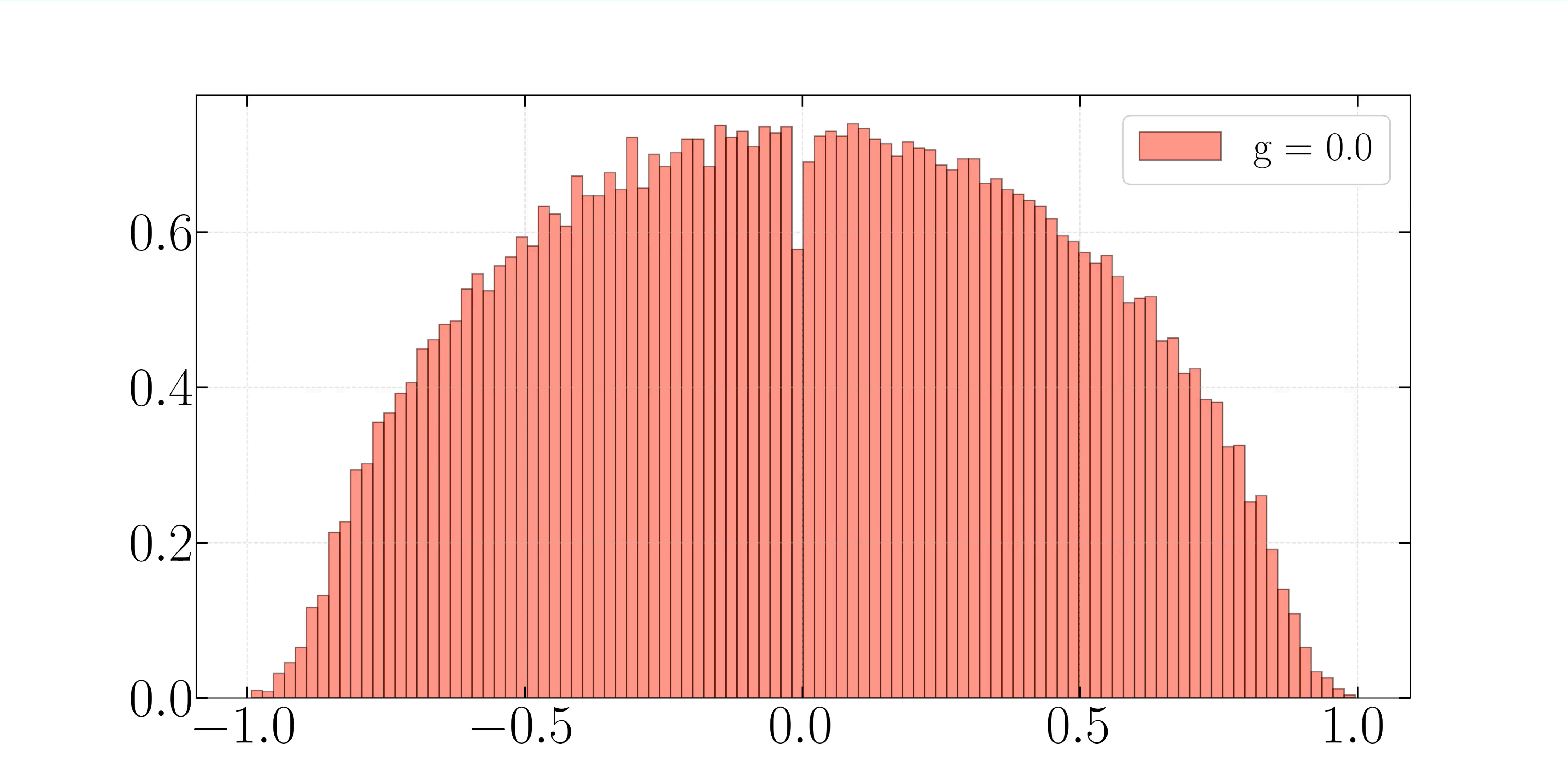
- We wanted to study the transition to complexity of the SYK-4 model
- We look at there interpolated version of the SYK-2 (disorders free fermions) and SYK-4

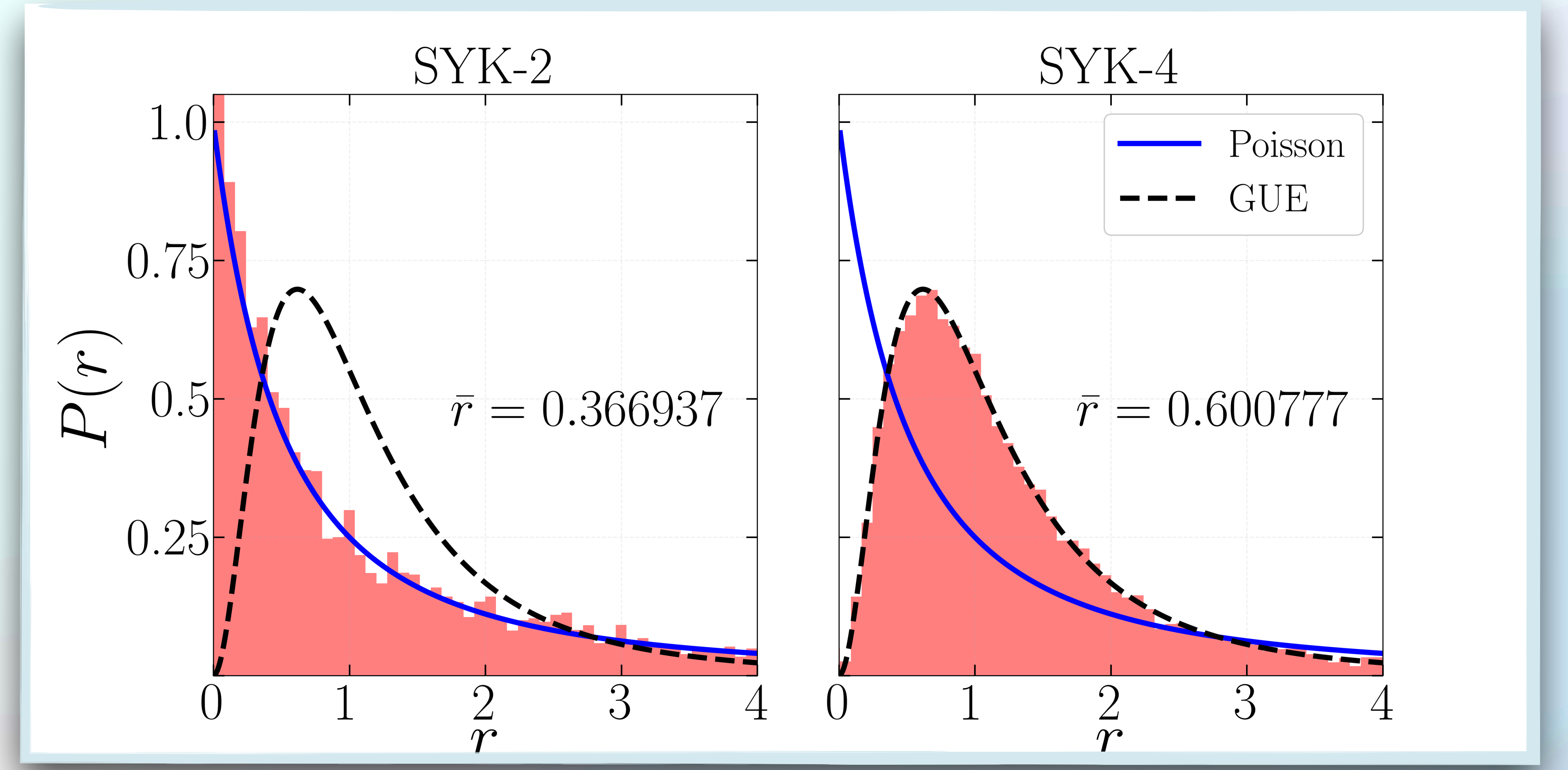
$$H(g) := (1-g)H^{\mathrm{SYK-4}} + gH^{\mathrm{SYK-2}},$$

- Both models admit a **volume law** in entanglement —> entanglement growth with the volume of the subsystem bipartition
- Non-disordered local Hamiltonians admit **area law**—> entanglement growth with the area of subsystem bipartition

Interpolated SYK

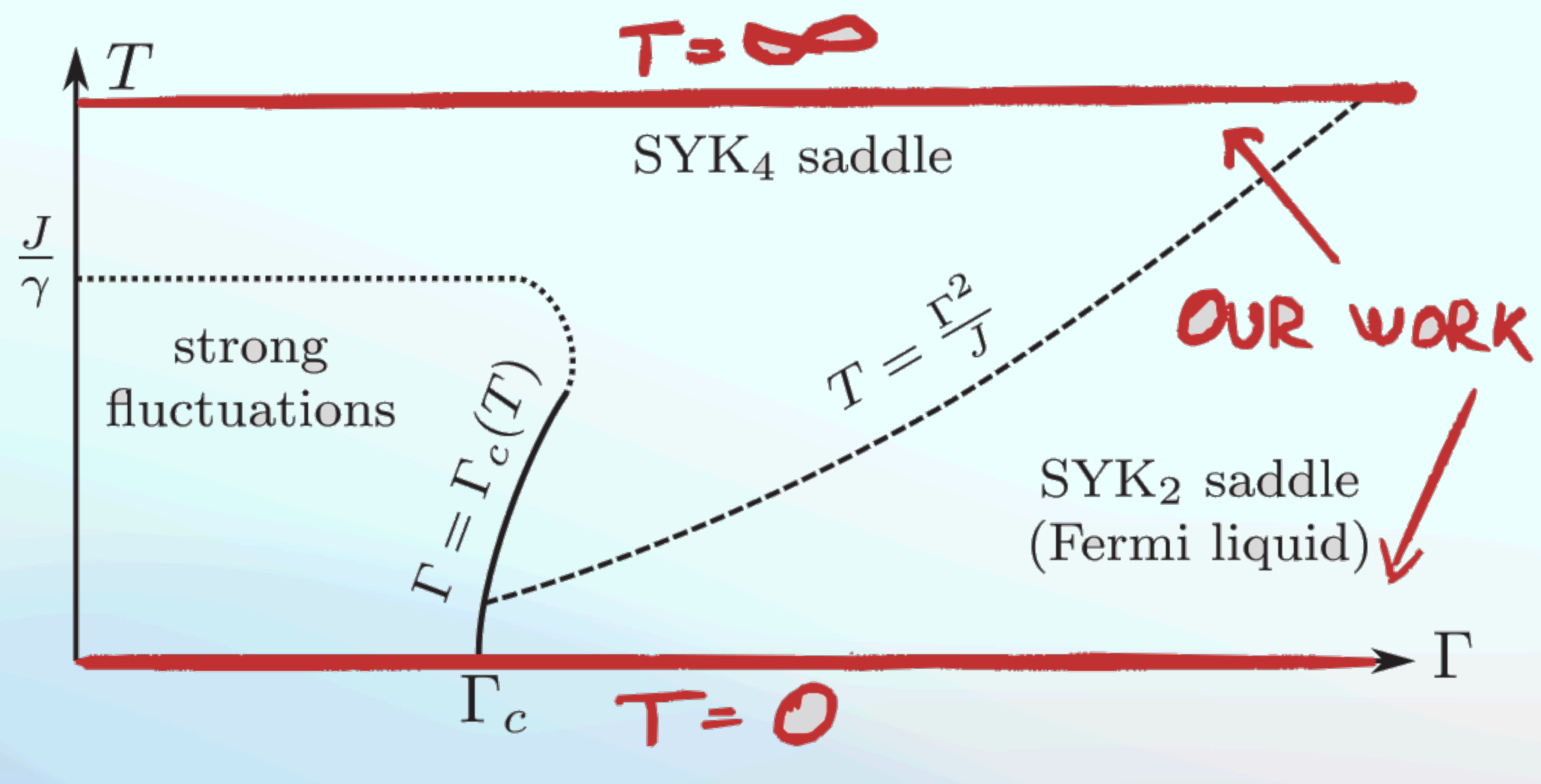
Density of States (DOS) and the Hamiltonian spectral statistics



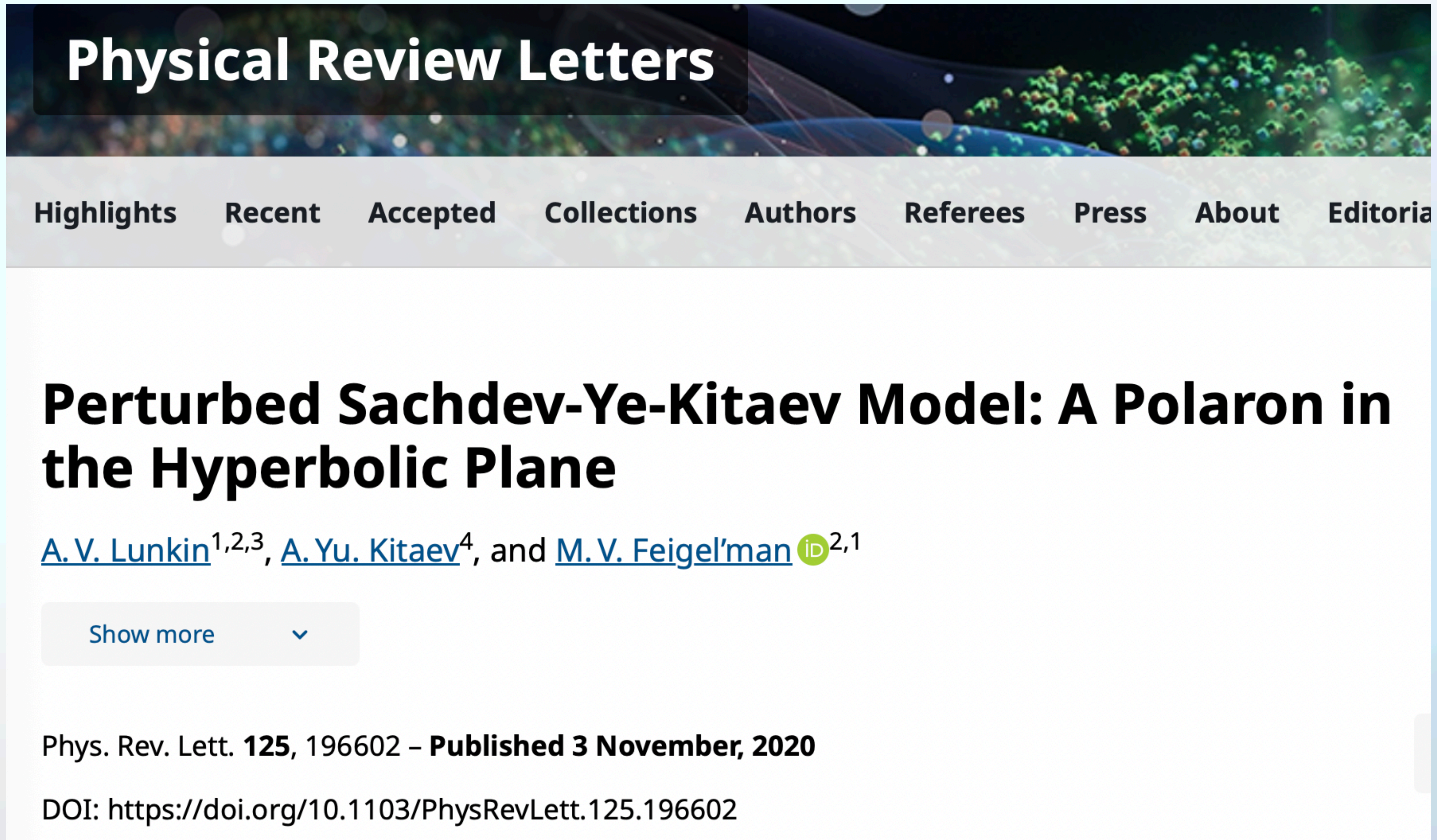


The context

We demonstrated using QI quantities the robust of different phases depending on the temperature and regime



Results of the paper



Physical Review Letters

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Perturbed Sachdev-Ye-Kitaev Model: A Polaron in the Hyperbolic Plane

A. V. Lunkin[1,2,3], A. Yu. Kitaev[4], and M. V. Feigel'man[2,1]

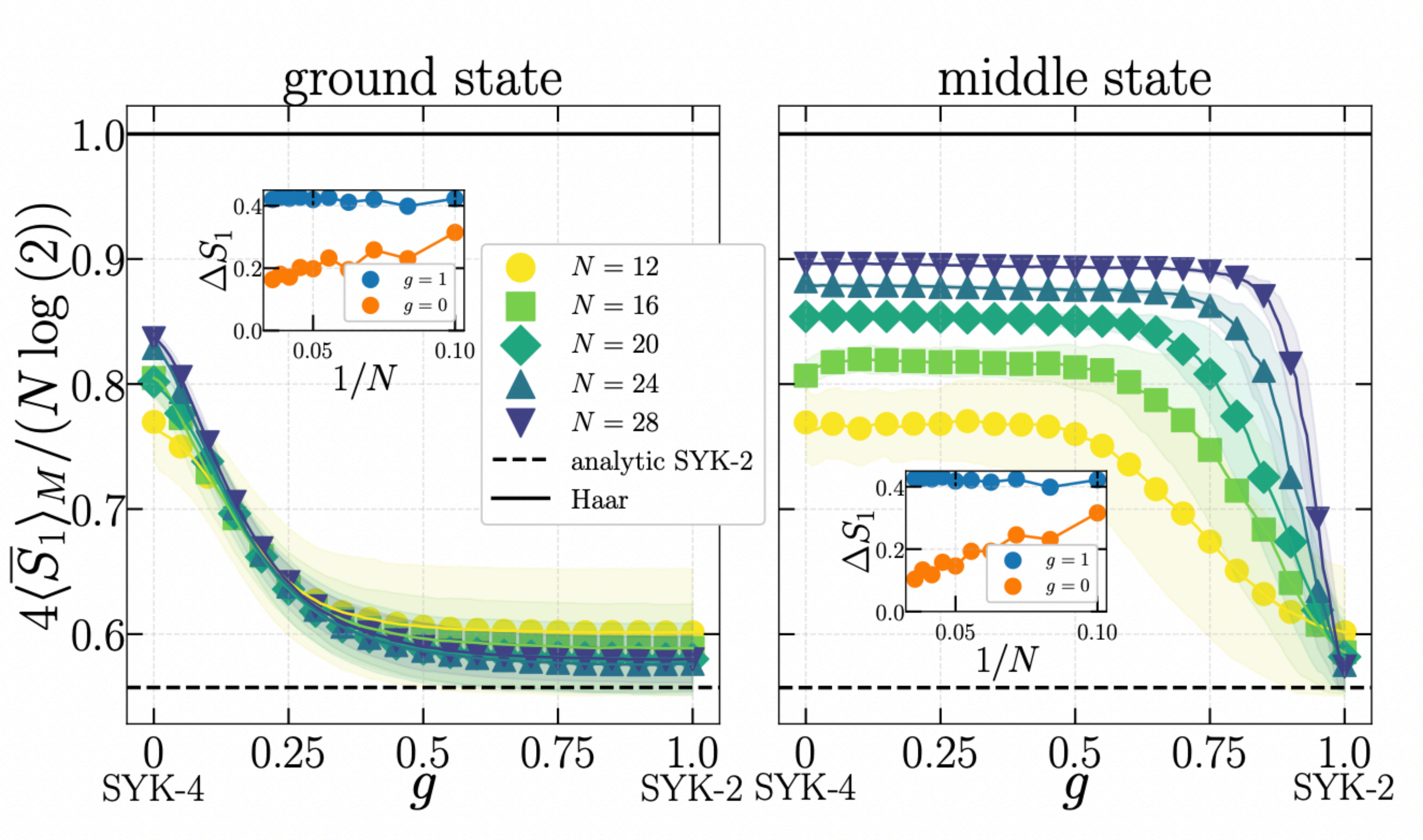
Show more

Phys. Rev. Lett. **125**, 196602 – **Published 3 November, 2020**

DOI: https://doi.org/10.1103/PhysRevLett.125.196602

Interpolated SYK

Von-Neumann entanglement entropy



Averaged bipartite entanglement in the GS (left panel) and MS (right panel) of the SYK4+SYK2 model. Shaded areas represent the standard deviation across M realizations of the Hamiltonian. We define the relative gap with respect to the the Haar (Page) value. The inset shows the finite size scaling of the relative gap.

$$S_1(\rho_R) = -\,\mathrm{Tr}(\rho_R \log(\rho_R))$$

$$\frac{2S_1^{\mathrm{Haar}}}{N\ln(2)} = 2f, \quad \text{for} \quad f \in [0,1/2],$$

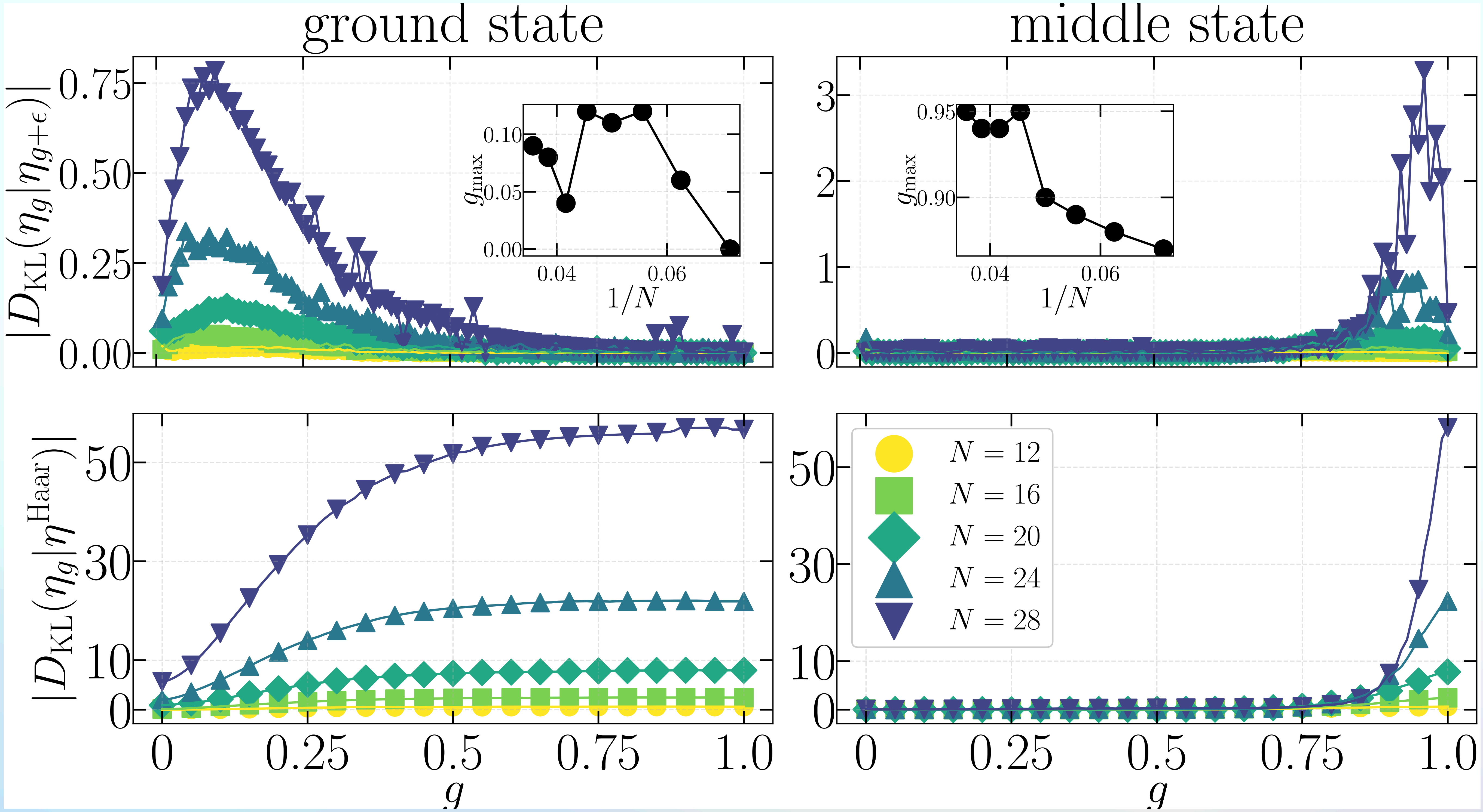
$$S_1^{\mathrm{SYK-2}}\left(R,f\right) = K(f)\ln(2)R,$$

$$K(f) = \left[1 - \frac{1+f^{-1}(1-f)\ln(1-f)}{\ln 2}\right].$$

Interpolated SYK

Reduced Density Matrix transition in complexity

$$\rho_R = \mathrm{Tr}_{\bar{R}}[\rho], \quad \rho = |\psi\rangle\langle\psi|$$



Renormalized reduced density matrix (RDM) eigenvalues of f = 1/2 subsystem-to-system.
The reference value is the Marchenko-Pastur (M-P) distribution.

Kullback-Leibler divergence

$$D_{KL}(p\,||\,q) := \sum_i p_i(\log p_i - \log q_i)$$

$$\eta^{\mathrm{Haar}}(x) = 1 - \frac{2}{\pi}\left(x\sqrt{1-x^2} + \arcsin x\right)$$

Interpolated SYK

Stabilizer entropy

N	16	18	20	22	24	26	28	30	32
Class	GOE	GUE	GSE	GUE	GOE	GUE	GSE	GUE	GOE

- **GOE** real symmetric matrices :

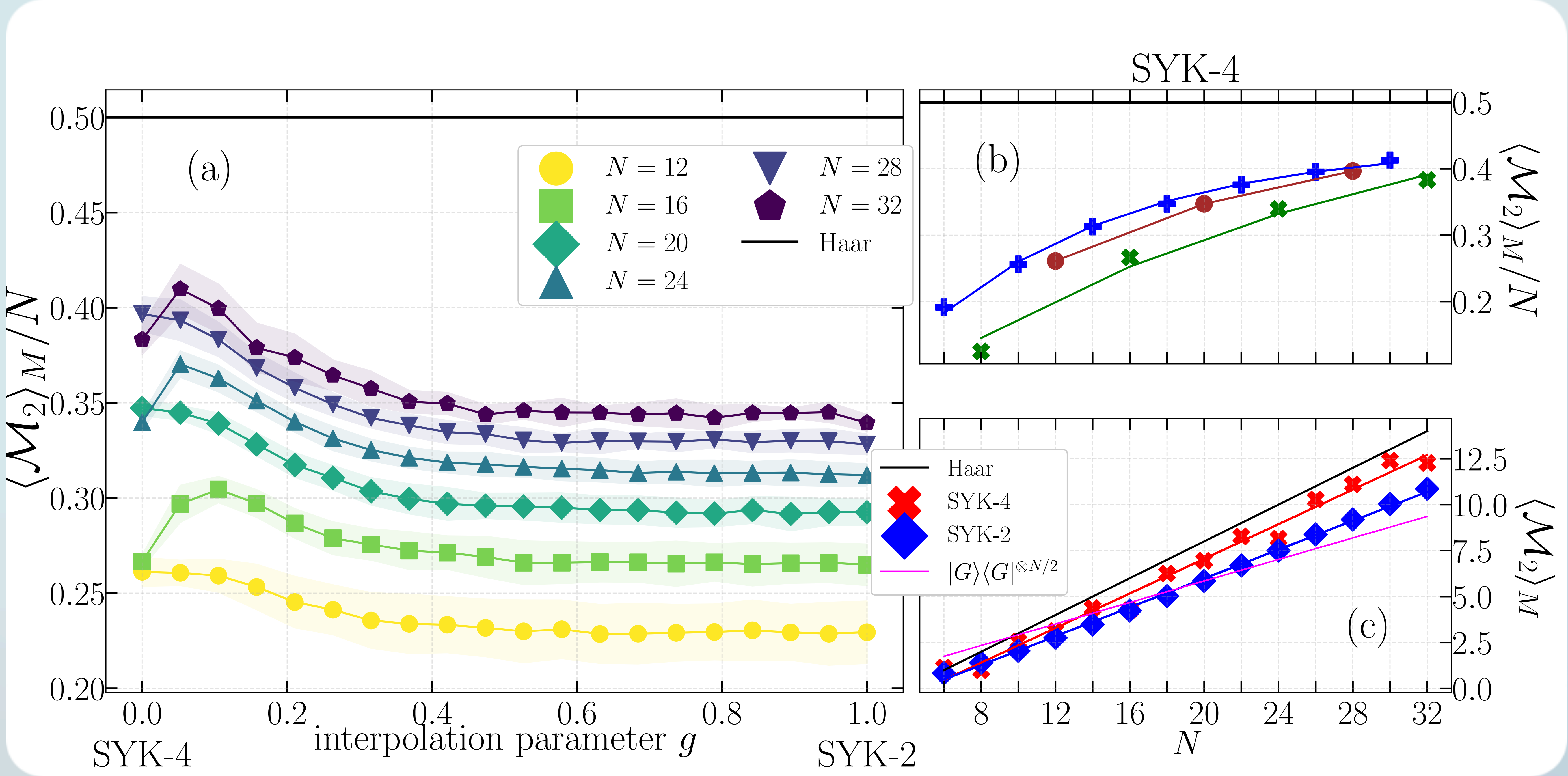
$$P(s) = \frac{\pi}{2} s e^{-\frac{\pi}{4}s^2}$$

- **GUE** hermitian matrices whit complex elements :

$$P(s) = \frac{32}{\pi^2} s^2 e^{-\frac{4}{\pi}s^2}$$

- **GSE** quaternionic matrices :

$$P(s) = \frac{2^{18}}{3^6\pi^3} s^4 e^{-\frac{64}{9\pi}s^2}$$



"Black holes and random matrices" J. S. Cotler, et al. Journal of High Energy Physics 2017, 118 (2017).

$$|G\rangle\langle G| = \frac{1}{2}\left(I + \frac{X+Y+Z}{\sqrt{3}}\right)$$

Interpolated SYK

Extrapolated behavior

$$M_2^{\mathrm{SYK}-4,\mathrm{GS}} \sim -2.4 + 0.95\frac{N}{2}\,.$$

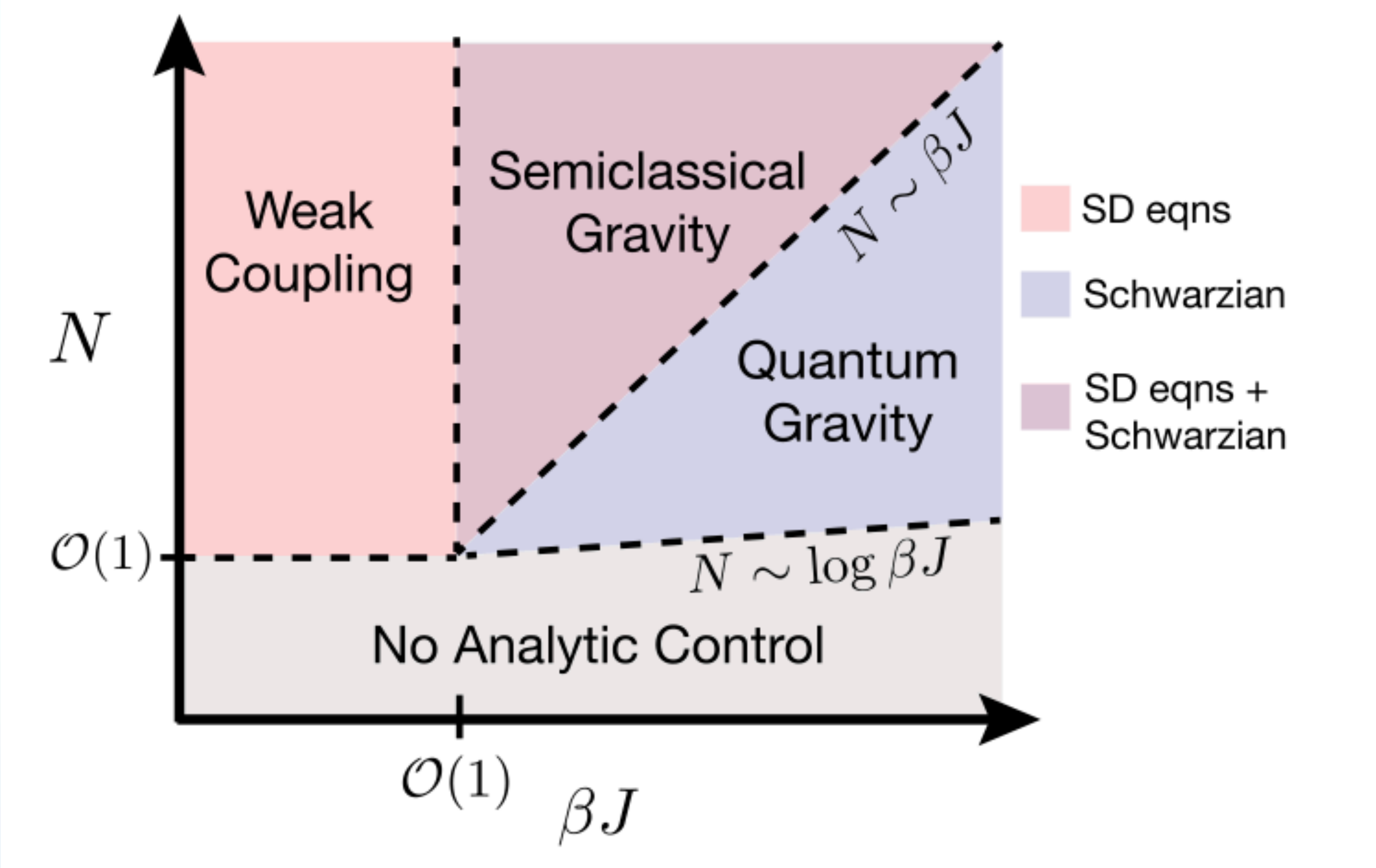
$$M_2^{\mathrm{Haar}} = -2 + \frac{N}{2},$$

$$M_2^{\mathrm{SYK}-4,\mathrm{MS}} \sim -2.6 + 0.96\frac{N}{2},$$

Stabilizer entropy is identify this model not as fully quantum chaotic and universal!

Our very natural question on SYK

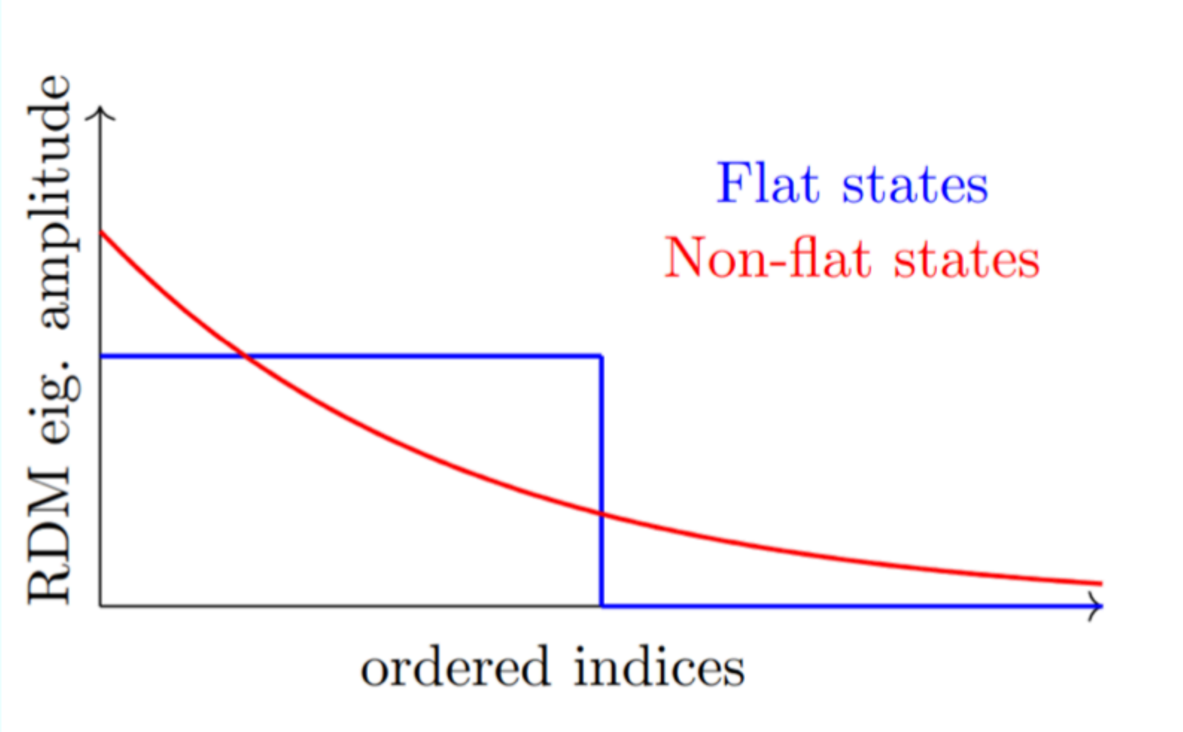
- Open problem: is there a way to approach computing stabilizer entropy from the gravity side?



Solvable limits of the SYK-4 model - PRL **126**, 030602 (2021)

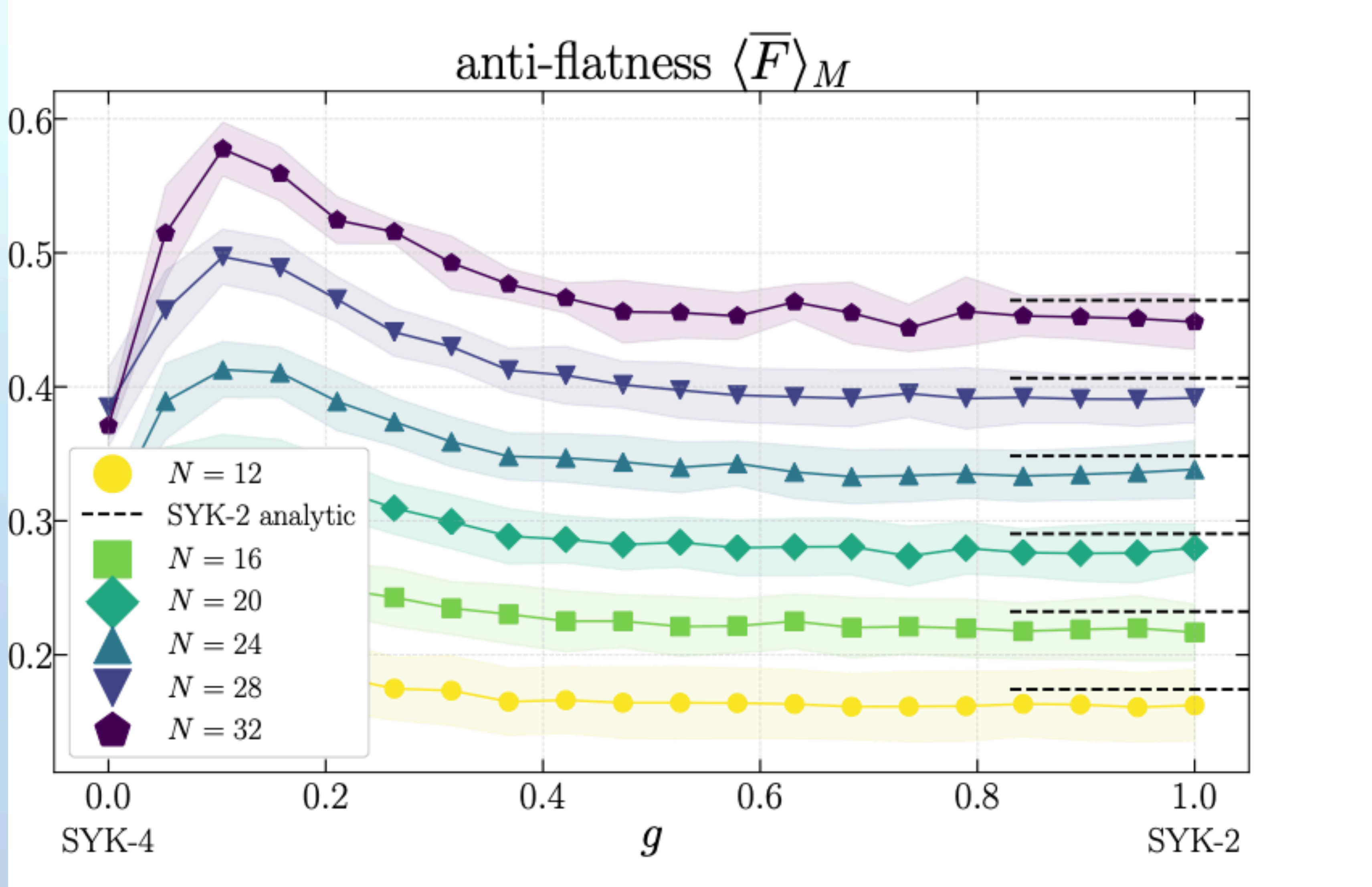
Interpolated SYK

Non-local magic



- Anti-flatness of RDM spectrum—> witness for non-local magic $\mathrm{F}(\rho_R) := 2\left(S_2(\rho_R) - S_3(\rho_R)\right)$

$$\mathrm{F}^{\mathrm{SYK-2}}(R,f) = 2R(1-f)\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{1}{2^n} - \frac{1}{2}\frac{3^n}{4^n}\right)\times {}_2F_1\left(\frac{1}{2}, 1-n, 2, 4f(1-f)\right)$$



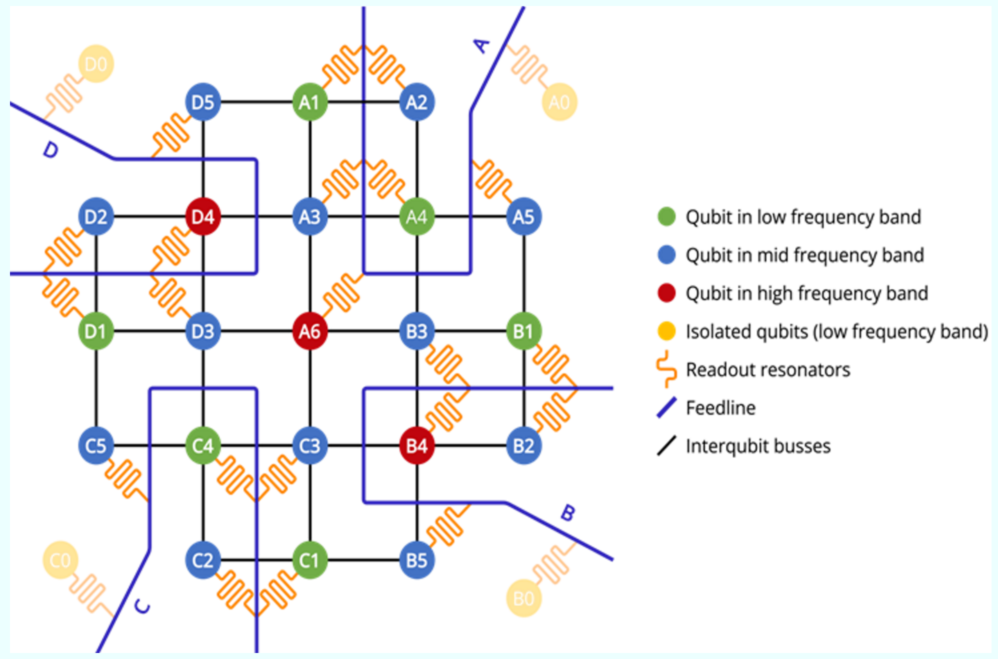
$$\mathrm{F}\left(\psi^{\mathrm{Haar}}_{N/2}\right) = \log\left(\frac{5}{4}\right).$$

Experimental measurements of non-local magic

Collaboration with the experimental group at University of Naples, which hosts Italy's first quantum computing chip. 25-qubit transmon qubit chip in operation

What is non-local magic?

- Non-local magic appears when magic gets **spread around** in a state with entanglement.
- Protocol: preparing non-local states and trying to **erase it** locally.



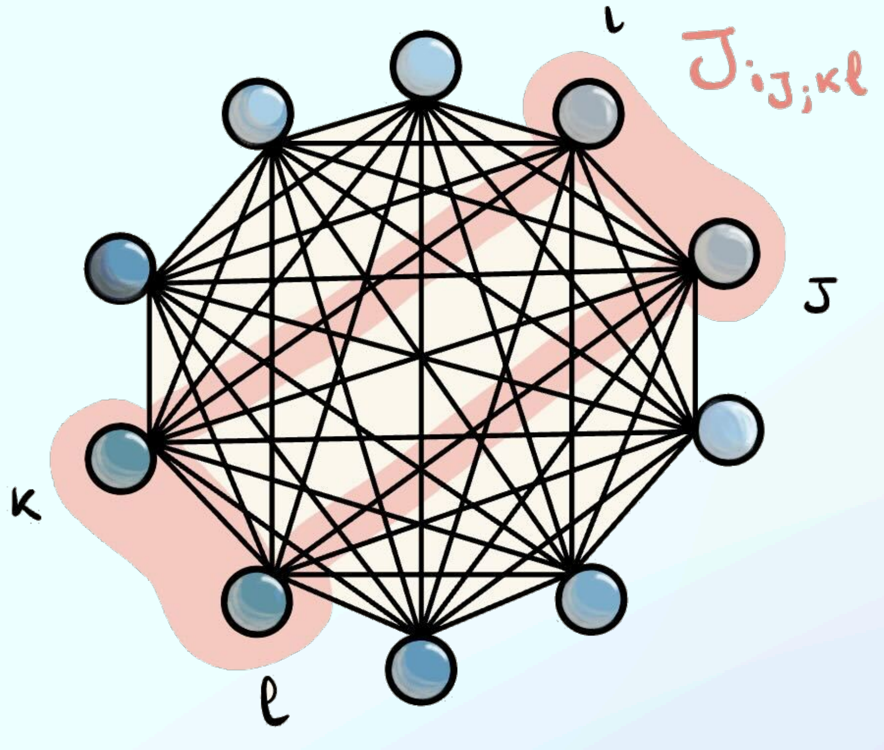
Theory group of Prof. Hamma

Experimental group of Prof. Tafuri

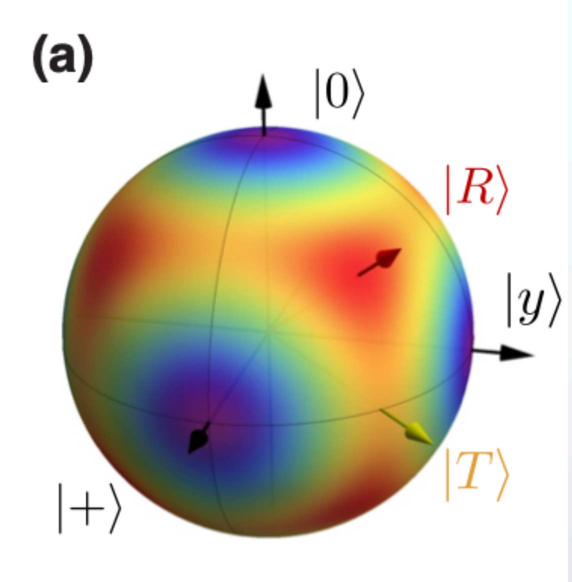
Quantum TechnologiesLab@UniNA

Checklist status

- Introduction to non-stabilizerness ✓
- Sachdev-Ye-Kitaev model ✓
- Spectral Form Factor and random walk ✗



SYK-4



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Spectral Form Factor (SFF) (arXiv:2505.05199)

Another tool for detecting Quantum Chaos

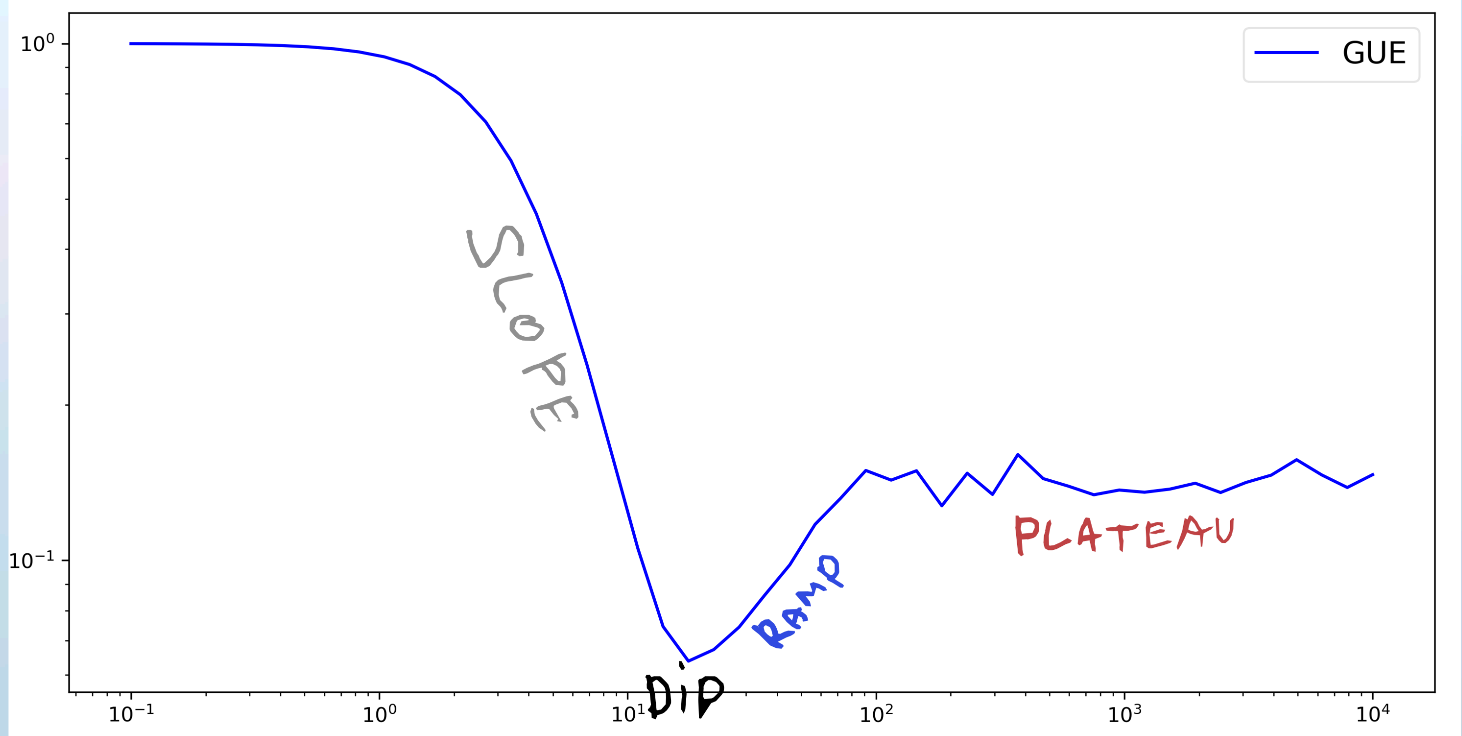
The SFF is a valuable tool to probe discrete level spectrum in quantum chaotic systems. It is defined via the partition function

$$Z(\beta) = \sum_n e^{-\beta E_n} \qquad Z(\beta + it) = \sum_n e^{-\beta E_n} e^{itE_n}$$

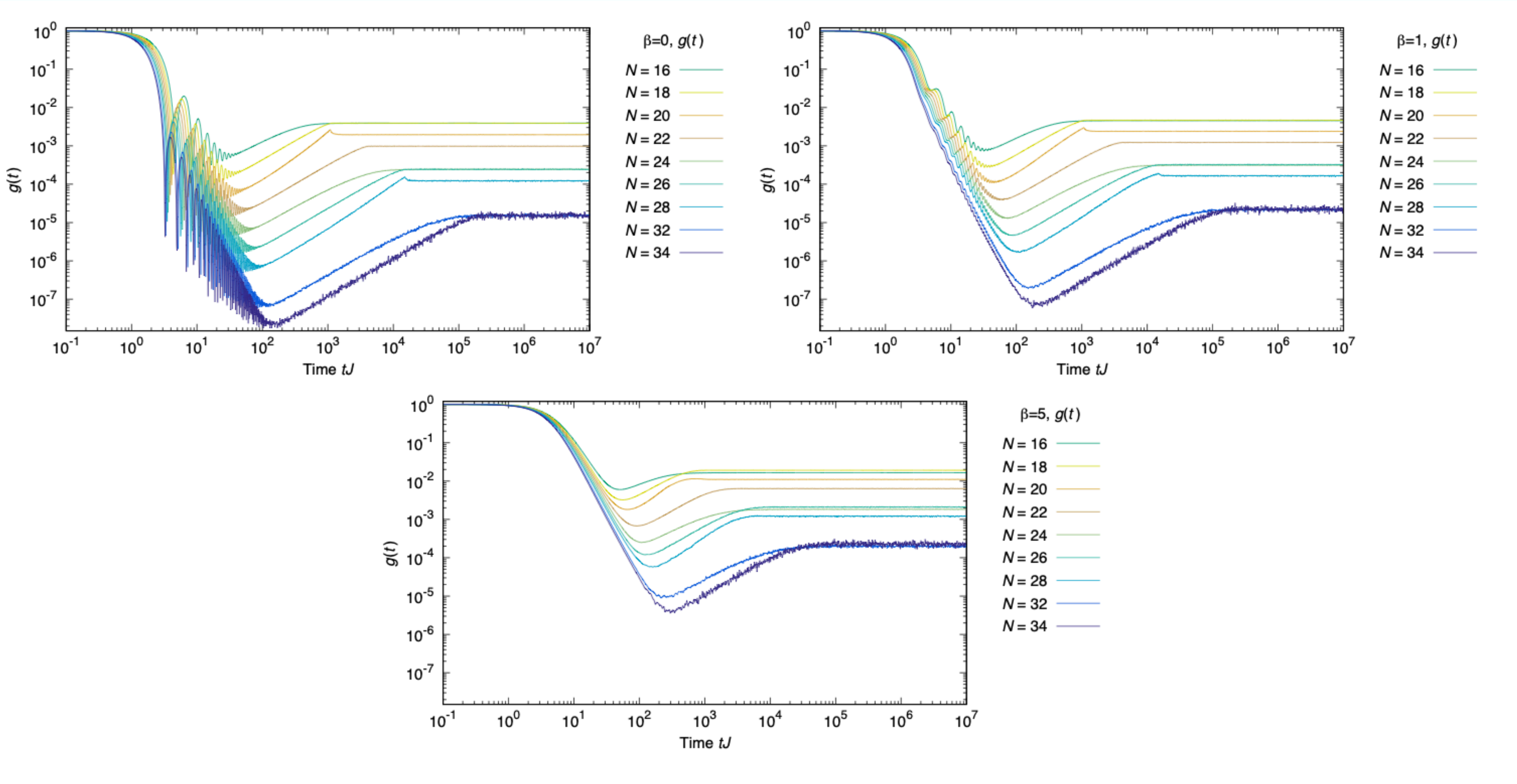
$$|Z(\beta + it)|^2 = \sum_{n=1}^{d} \sum_{m=1}^{d} e^{-\beta(E_m+E_n)} e^{i(E_m-E_n)t},$$

In quantum chaotic (non-integrable) systems whose spectrum is typically described by Random Matrix Theory (RMT) ensembles the SFF displays **slope-dip-ramp-plateau** behavior.

In non-chaotic (integrable) systems, the SFF has a **slope** and a **plateau**, but no linear ramp.



SFF in SYK-4



J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, “Black holes and Random Matrices” Journal of High Energy Physics 2017 118 (2017)

Our work

Long-time limit of the SFF

$$\overline{f(t)} := \lim_{T\to\infty} \frac{1}{T}\int_0^T dt f(t)\,, \qquad \overline{|Z(\beta+it)|^2} = \sum_{n,m} \lim_{T\to\infty} \frac{1}{T}\int_0^T \mathrm{d}t e^{-it(E_n-E_m)},$$

The higher moments:

$$|Z(\beta+it)|^{2M} = \sum_{n_1=1}^{d}\sum_{n_2=1}^{d}\cdots\sum_{n_M=1}^{d}\sum_{m_1=1}^{d}\sum_{m_2=1}^{d}\cdots\sum_{m_M=1}^{d} e^{-\beta\sum_{j=1}^{M}(E_{n_j}+E_{m_j})} e^{-it\sum_{j=1}^{M}(E_{n_j}-E_{m_j})}$$

$$M = 1 \to E_{n_1} = E_{m_1}$$

$$M = 2 \to E_{n_1} = E_{m_1}, E_{n_2} = E_{m_2}; E_{n_1} = E_{m_2}, E_{n_2} = E_{m_1}$$

$$M = 3 \to 6 \quad conditions$$

$$M = \dots$$

Numerical observation at first

Generalized code

```
In[•]:= SetTamponBasedOnPermutations[n_] :=
  Module[{perms, alphas, betas, result}, perms = Permutations[Range[n]];
   alphas = Table[Symbol["α" <> ToString[i]], {i, n}];
   betas = Table[Symbol["β" <> ToString[i]], {i, n}];
   result = Null;
   Do[If[And @@ Thread[alphas == betas[[perms[[i]]]]], result = i],
    {i, Length[perms]}];
   result]
```

The OEIS is supported by the many generous donors to the OEIS Foundation.

0 1 3 6 2 7 13 20 12 21 11 22 10 23 OEIS THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

The On-Line Encyclopedia of Integer Sequences (OEIS)

Enter a sequence, word, or sequence number:

1,2,3,6,11,23,47,106,235

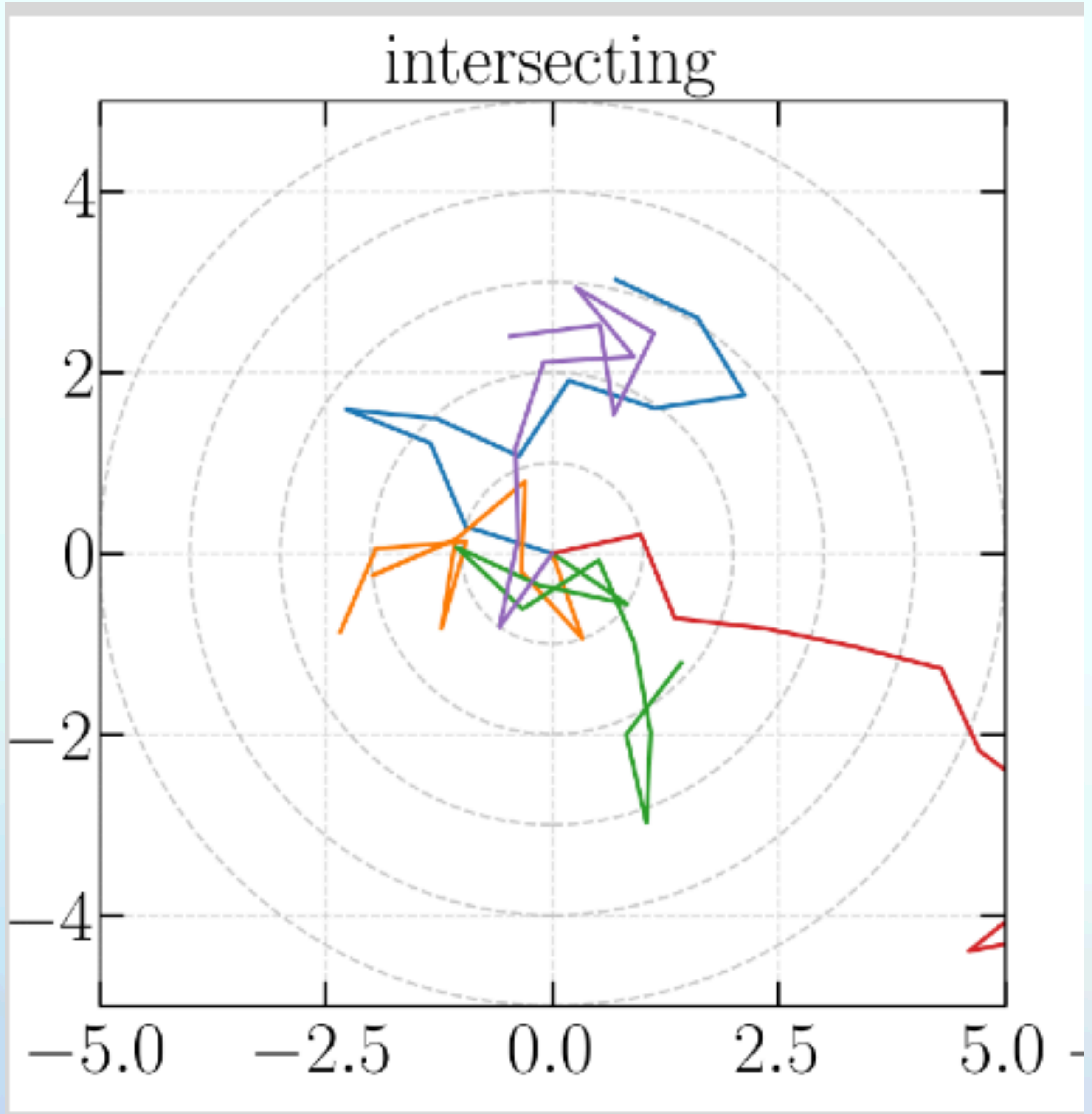
```
Case(m = 2); Total terms: 4096 => How many non-zero terms: 120
```

```
(* changing V we obtain *)
V = {1, 2, 3, 4, 5, 6, 7, 8};
number = {1, 6, 15, 28, 45, 66, 91, 120};
```

Jonathan M. Borwein, Dirk Nuyens, Armin Straub and James Wan, Random Walk Integrals, The Ramanujan Journal, October 2011, 26:109. DOI: 10.1007/s11139-011-9325-y.
Cesar Ceballos and Viviane Pons, The s-weak order and s-permutahedra II: The combinatorial complex of pure

Planar random walks

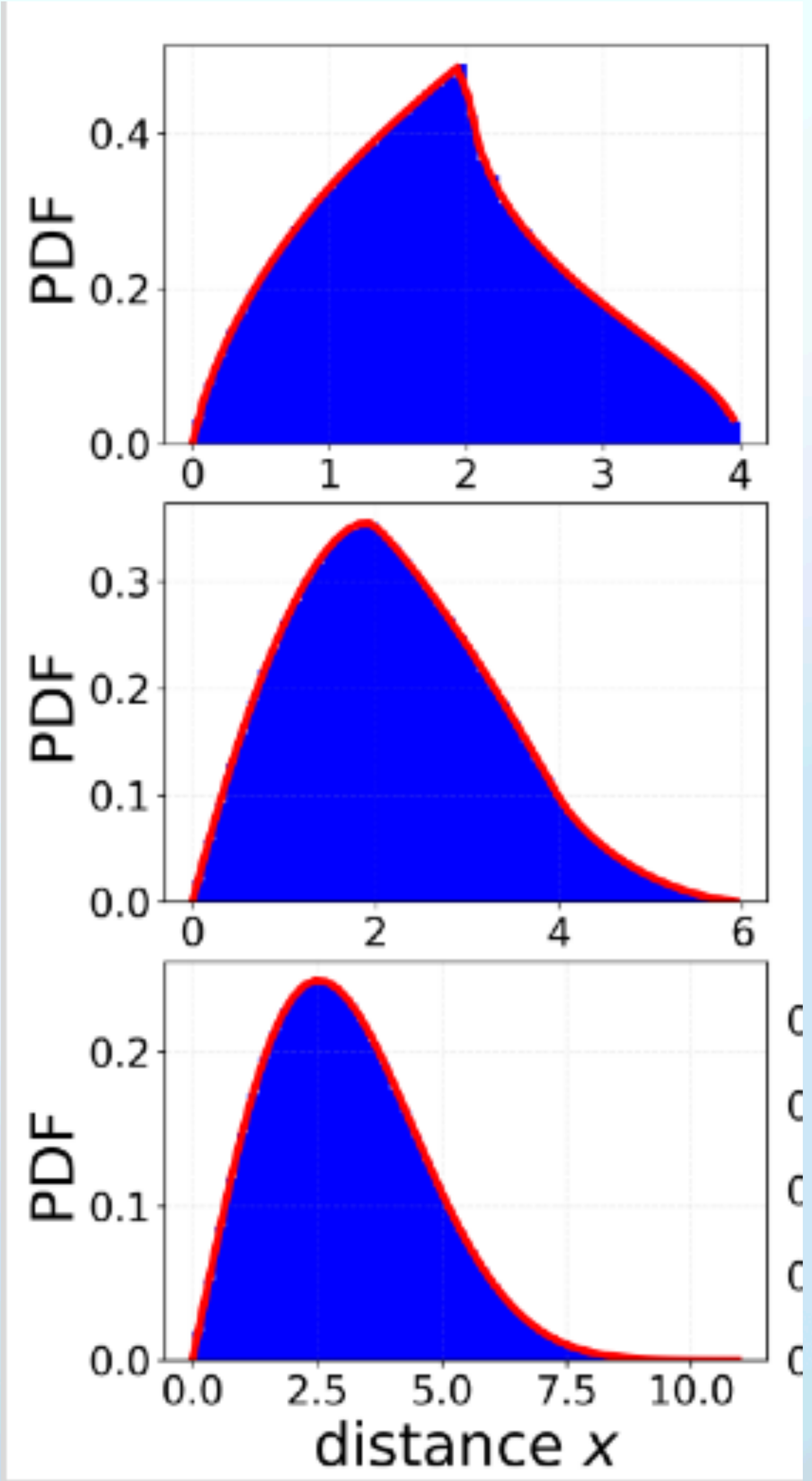
The mathematical problem of determining where a random walker ends up after n steps relative to their starting point has been thoroughly explored.



Computer simulations of 5 trajectories of length up 10

$$\mathrm{PDF}(x,n) = \int_0^\infty J_0(xt)\left[J_0(t)\right]^n xt \ \mathrm{d}t$$

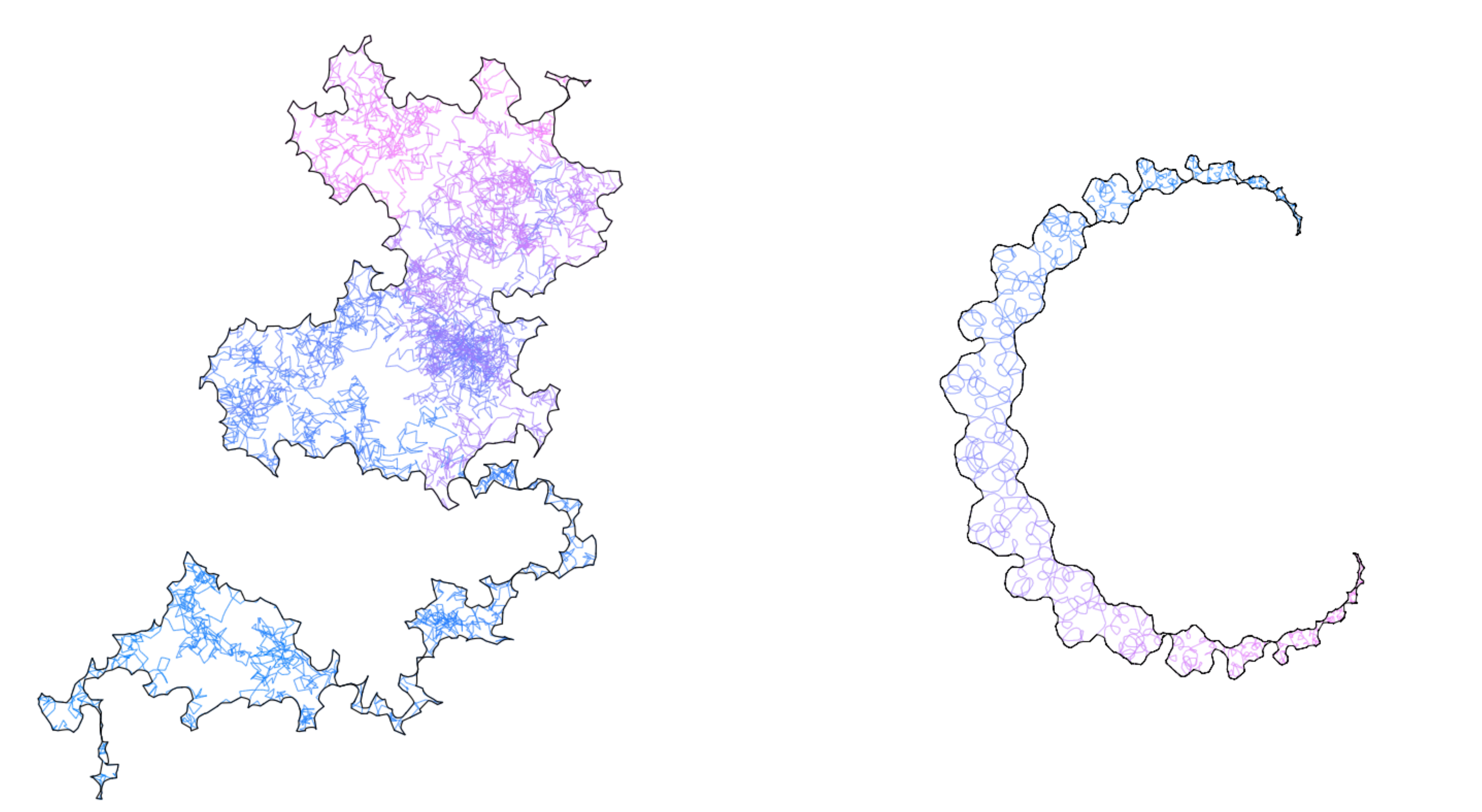
$$W_n(s) := \int_0^n \mathrm{PDF}(t,n)t^s \ \mathrm{d}t \quad \text{or} \quad W_n(s) := \int_{[0,1]^n} \left| \sum_{k=1}^n e^{2\pi i x_k} \right|^s \mathrm{d}\boldsymbol{x},$$



Where does random walker end up after n = 4,6,12 steps

So we started digging...

and put a rigorous mathematical framework



Fractals and their frontiers, in black, corresponding to physical models: non-integrable (left) vs integrable (right). Color from blue to pink corresponds to increasing time-steps of the random walks. The frontier is essentially the boundary of the fractal without the inner islands, see text for details. The non-integrable Hamiltonian is the XXZ model with next nearest neighbor interactions XXZ chain with (Δ,α) = (0.4,0.5). The integrable Hamiltonian is the XY model with parameters (h,γ) = (0.2,0.3);

	Quadratic	Bethe-Ansatz	Non-Integrable
Level spacing distribution	Poisson	Poisson	Wigner-Dyson
Normalized $\lvert\chi(t)\rvert^2$ distribution	LogNormal	Exp(1)	Exp(1)
Dimension of frontier	1.01 ± 0.12	1.24 ± 0.08	1.32 ± 0.08

Table I. Expectation of different metrics of chaos for increasingly more chaotic models (from left to right).

Derived an iterative close-form expression

In case of quantum chaotic spectra

$$I_p = \sum_{q=1}^{p-1} \binom{p-1}{q-1} \frac{p!}{(p-q)!} a_q X_q I_{p-q} + p! a_p X_p.$$

$$X_n := \sum_{j=1}^{N_B} (d_j)^{2n}$$

$$I_1 = X_1, I_2 = 2(X_1)^2 - X_2$$

$$I_3 = 6\,(X_1)^3 - 9 X_1 X_2 + 4 X_3\,.$$

Applied to the SYK

$$K_m = \frac{\mathsf{E}_x\left[\overline{|\chi(t)|^{2m}}\right]}{\left(\mathsf{E}_x\left[\overline{|\chi(t)|^{2}}\right]\right)^m} = \frac{\mathsf{E}_x\left[I_m\right]}{(\mathsf{E}_x[I_1])^m} \,,$$

	High Temperature $T = \infty$		Low Temperature $T = 0.01$	
	Gaussian (%)	Exact (%)	Gaussian (%)	Exact (%)
K_2	0.229	0.033	46.452	0.011
K_3	1.018	0.434	79.754	0.007
K_4	1.484	0.316	93.936	0.025

Table II. Relative errors (in percentages) of the moments of the SFF of the SYK-4 model between the exact formulae (average of) Eqns. (15) and the Gaussian approximation $K_m \simeq m!$. The numerical data are obtained by sampling t uniformly in $[10^5, 2\times10^5]$, 10^4 times and performing an additional ensemble average over 100 realizations for the SYK-4 model with $N = 18$ Majorana fermions. Increasing the number of realizations and time domain sampling leads to better agreement. The superior agreement of the exact expression is most evident for low temperatures.

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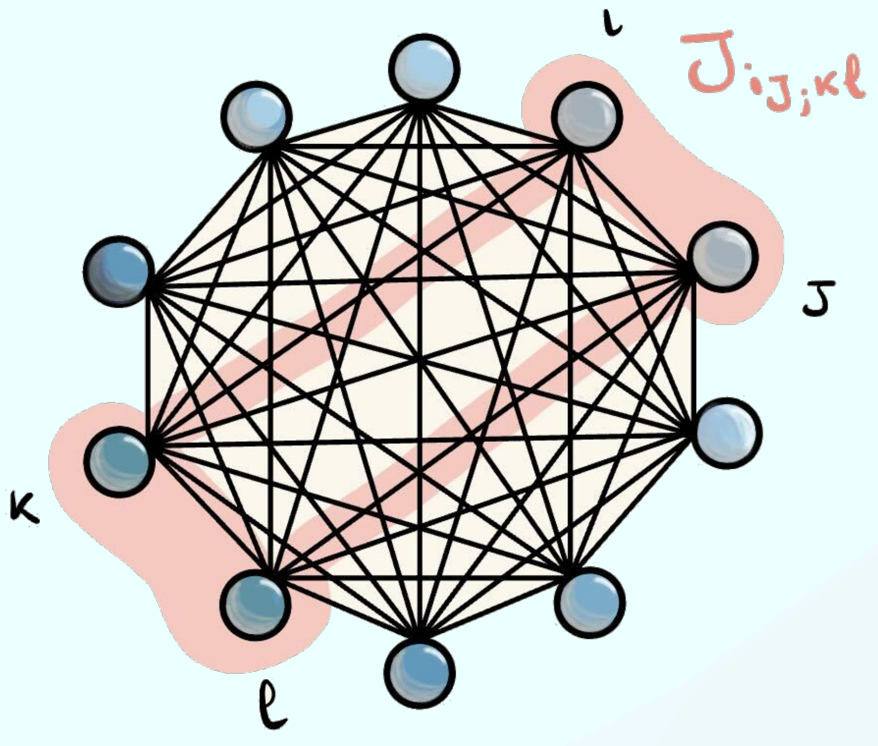
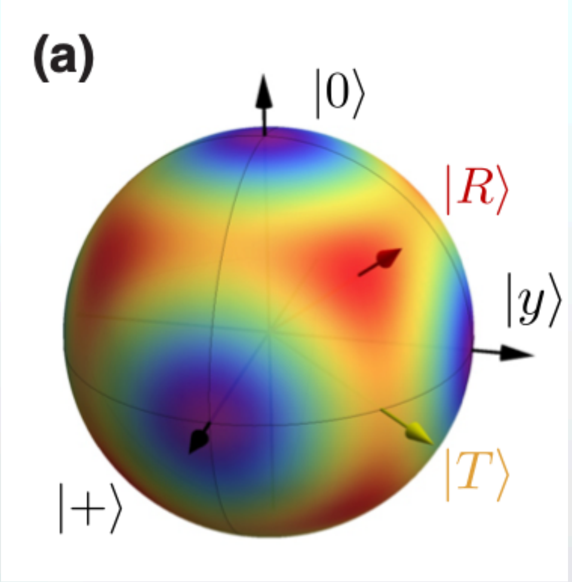


SYK-4



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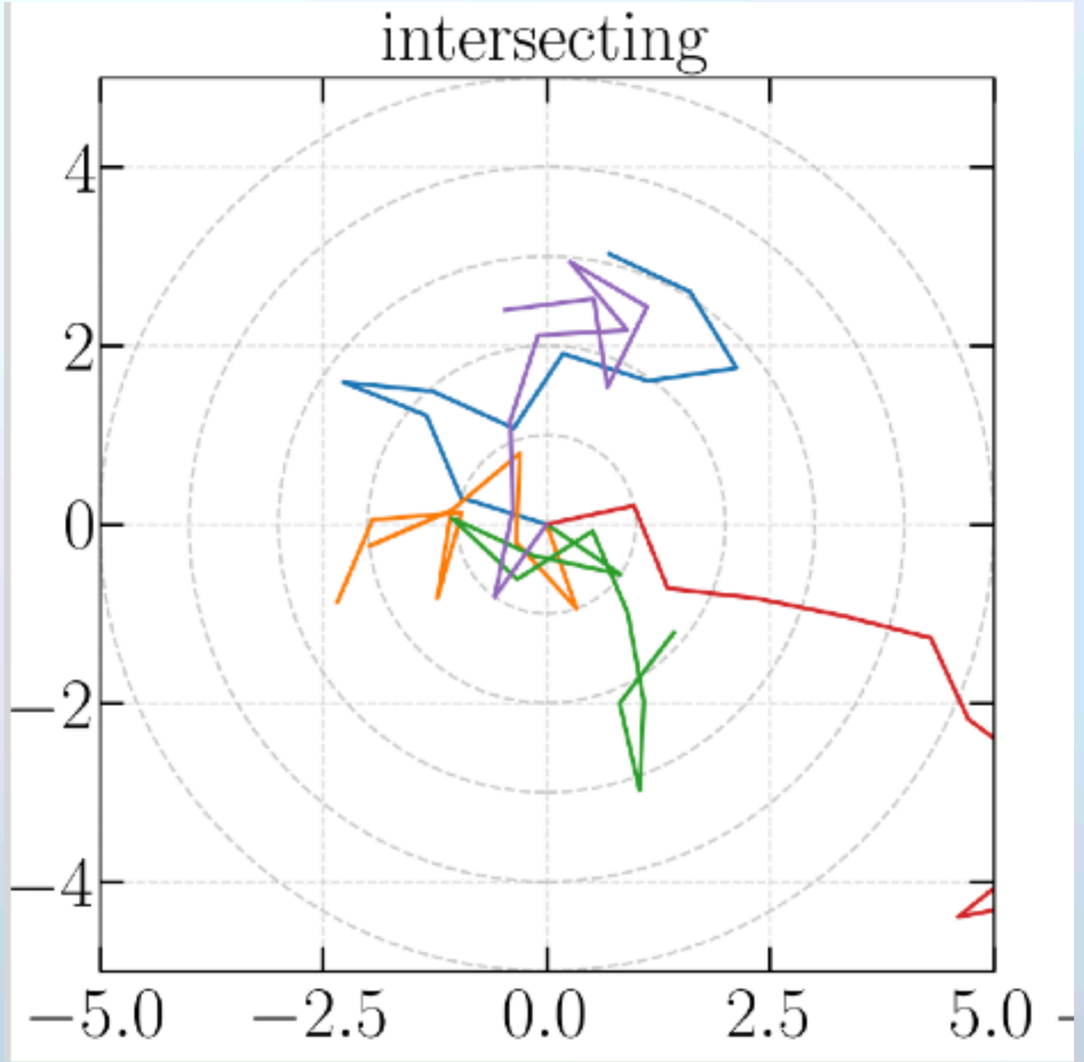

Thanks for the attention!

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