

Kvantni računari - teorija i eksperimenti

*ili

Ljubavna priča između teorije i eksperimenta
(Pragma (πραγμα): dugotrajna i postojana ljubav koja se razvija dugo vremena)



Teorija



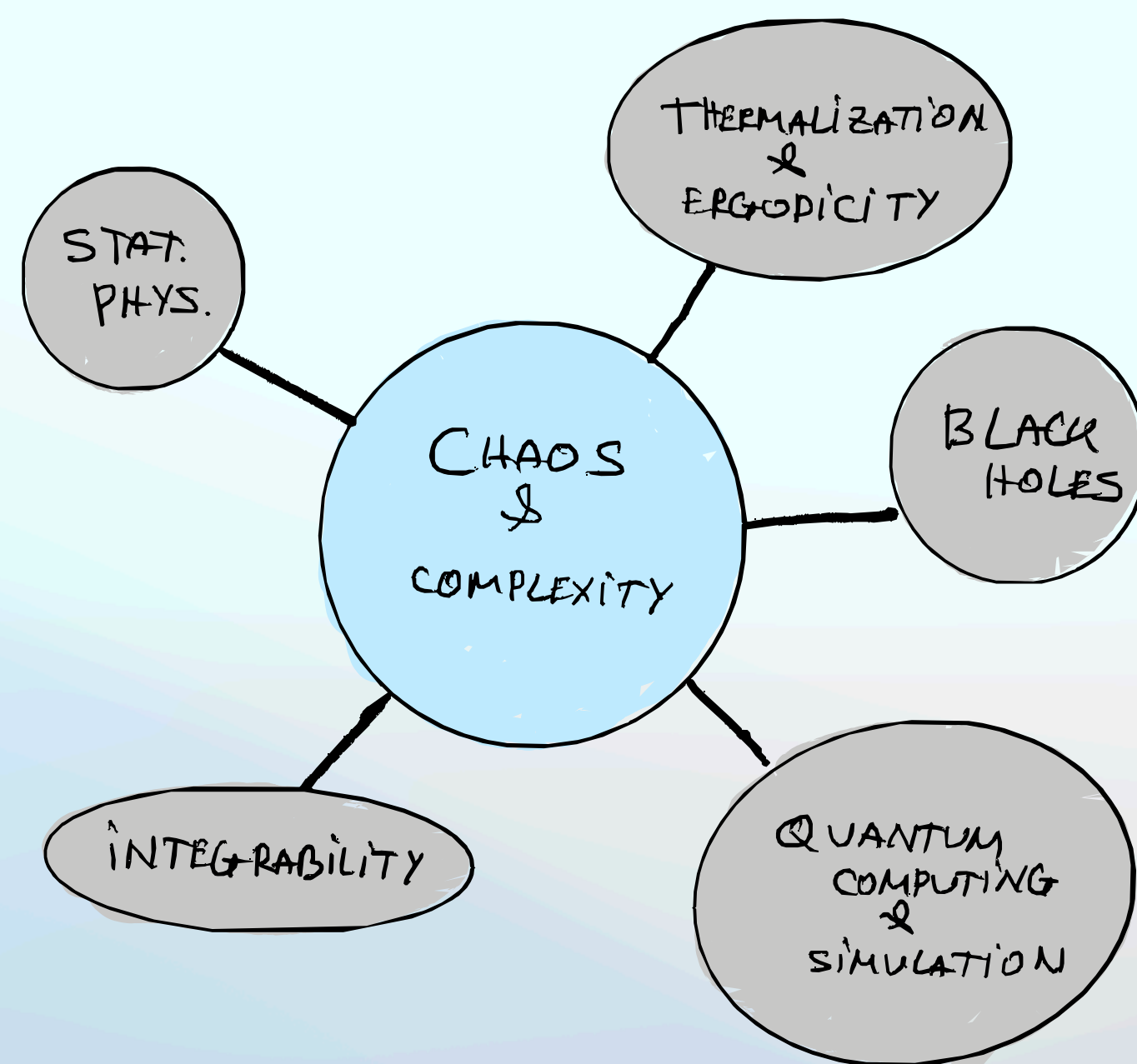
Eksperiment

Jovan Odavić 28/04/2026 - Novi Sad

arXiv: 2511.15576

Nešto ukratko o meni

- Statistička fizika i fizika višestrukih korelisanih sistema
- Teorija haosa i nelinearna dinamika
- Kvantna informatika



iz Novog Sada...

Vezuv 



...u Napulj

Kvantni računar u Napulju

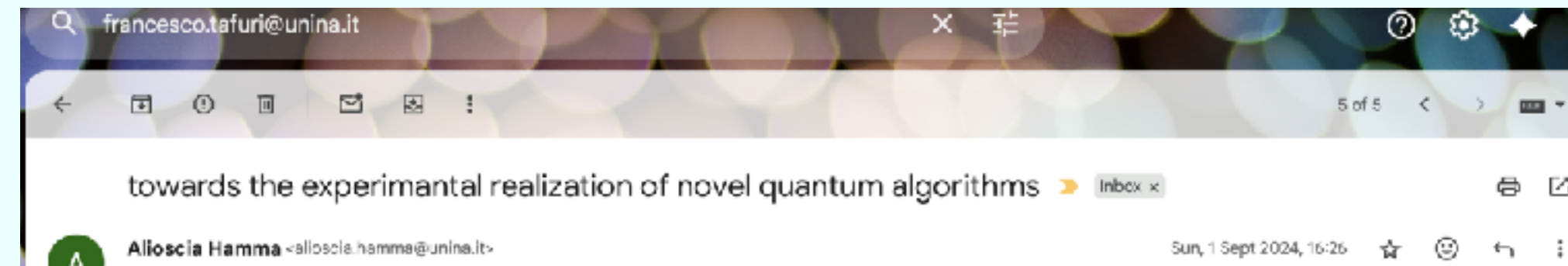
- Prvi kvantni računar u Italiji
- Imali smo privilegiju da radimo prve eksperimente + benchmarking
- Razvijaju se na univerzitetima i u velikim kompanijama (Google, Microsoft, Amazon, IBM)



prof. Tafuri na inauguraciji računara



Kolaboracija započeta pre više od godinu dana



Teorijska grupa prof. Hamma



Eksperimentalna grupa prof. Tafari

Prošlogodišnja Nobelova nagrada

... za otkriće makroskopskog kvantno mehaničkog efekta tunelovanja i kvantizaciju u električnim kolima

Nobel Prize in Physics 2025



Ill. Niklas Elmehed © Nobel Prize Outreach

John Clarke

Prize share: 1/3



Ill. Niklas Elmehed © Nobel Prize Outreach

Michel H. Devoret

Prize share: 1/3

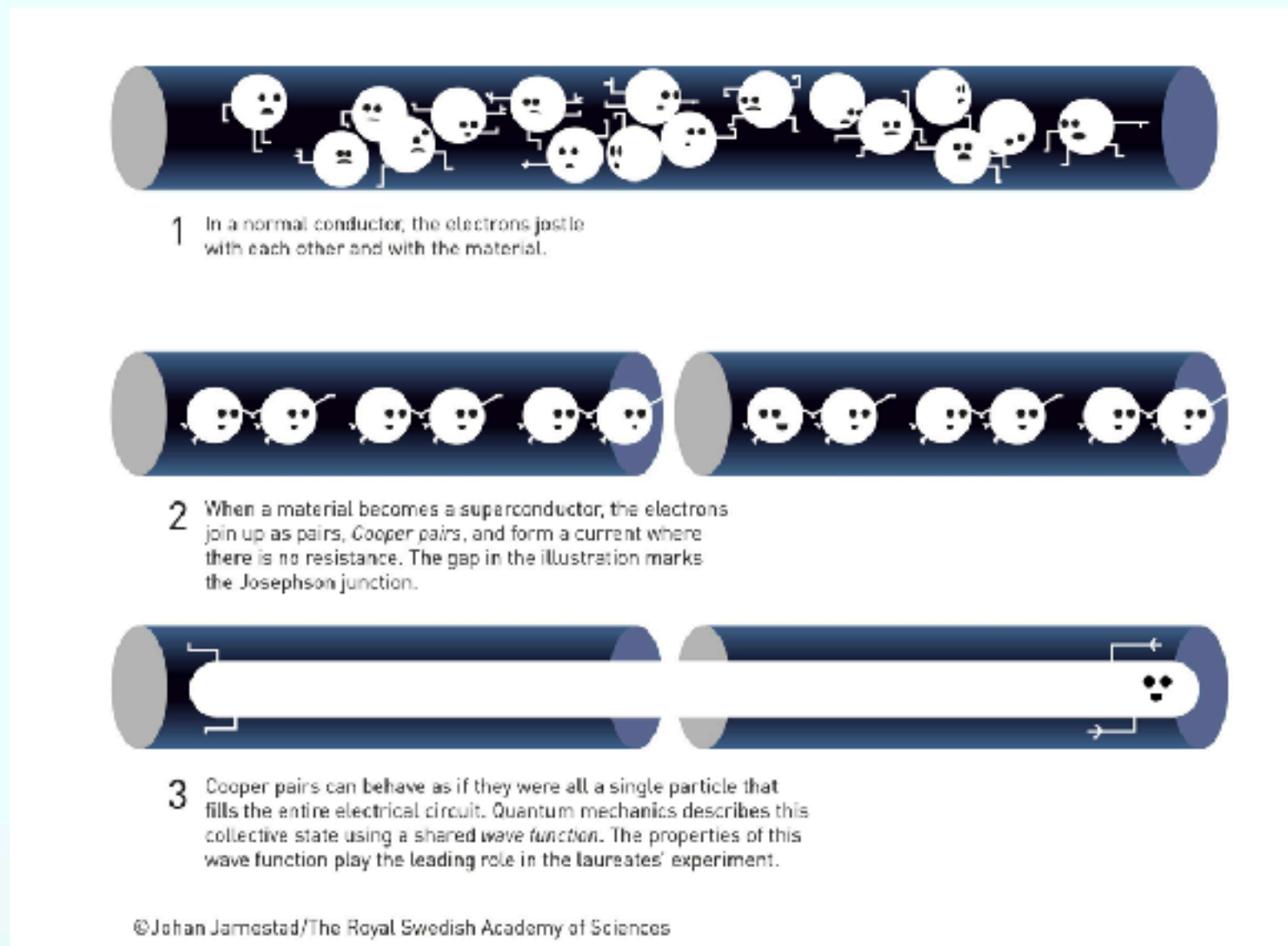


Ill. Niklas Elmehed © Nobel Prize Outreach

John M. Martinis

Prize share: 1/3

Ključni detalji superprovodnih qubit-a



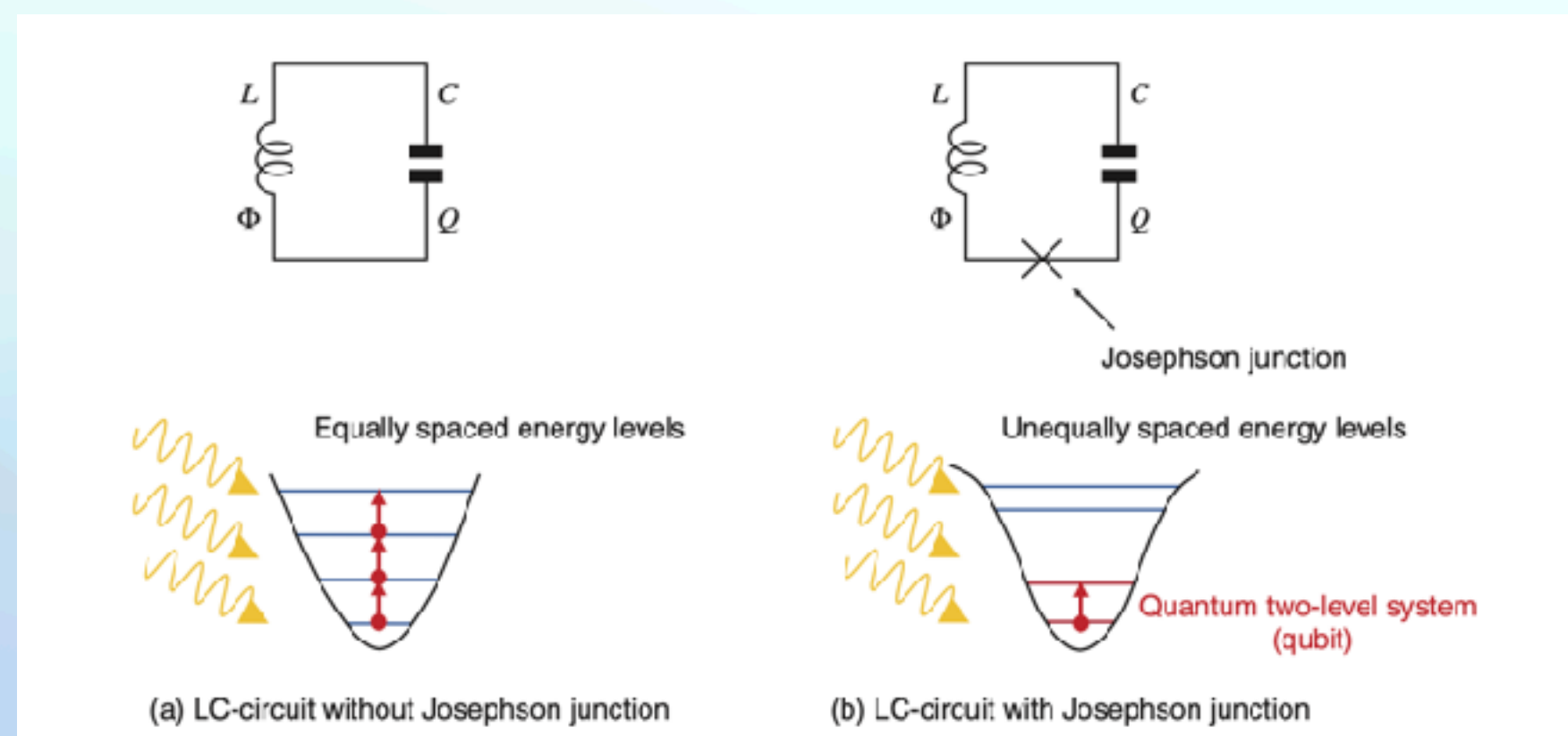
Poznati luster (CHANDELIER)



Napuljska laboratorija



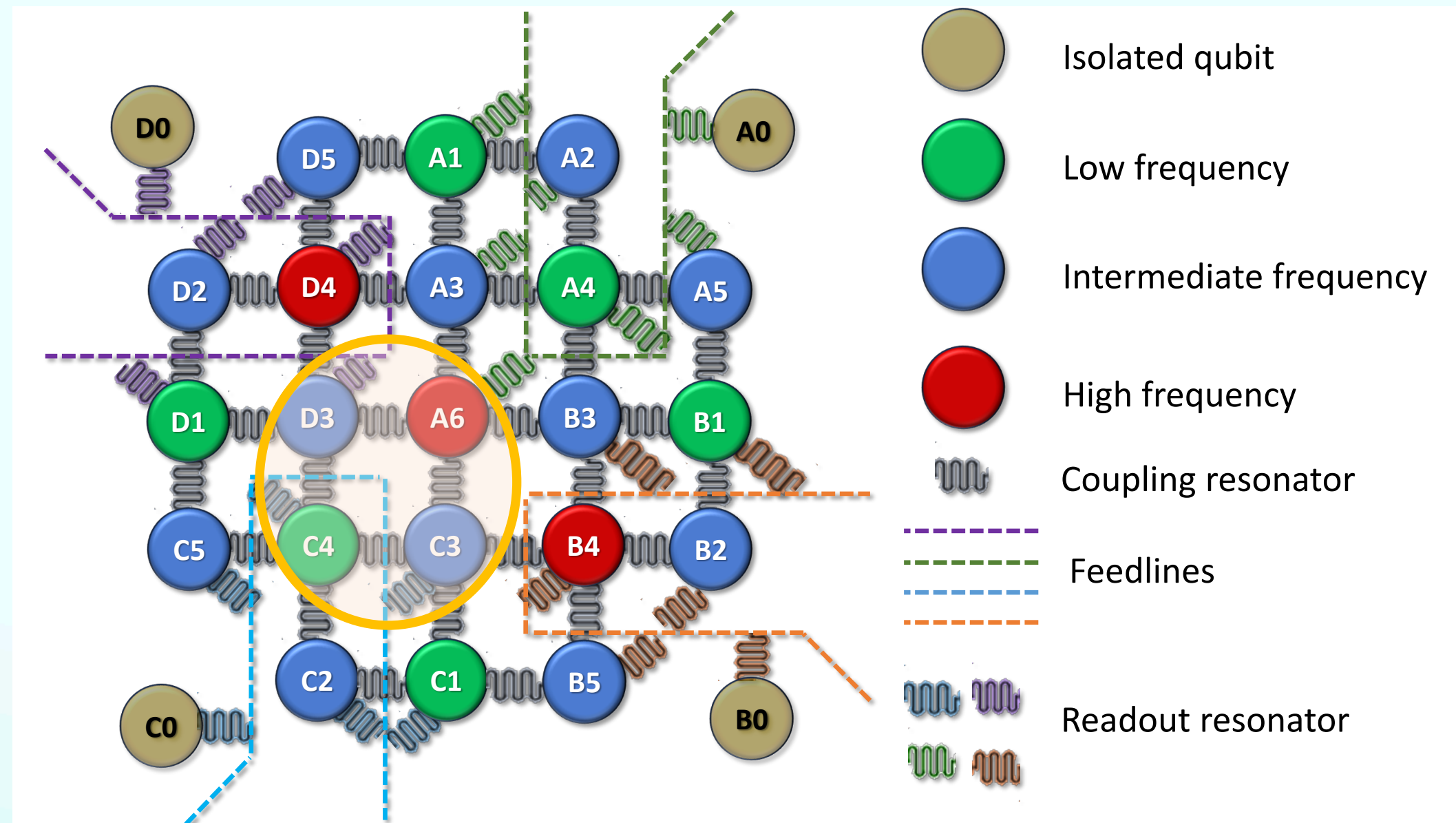
Kvantni efekti -> potrebna niska temperatura!



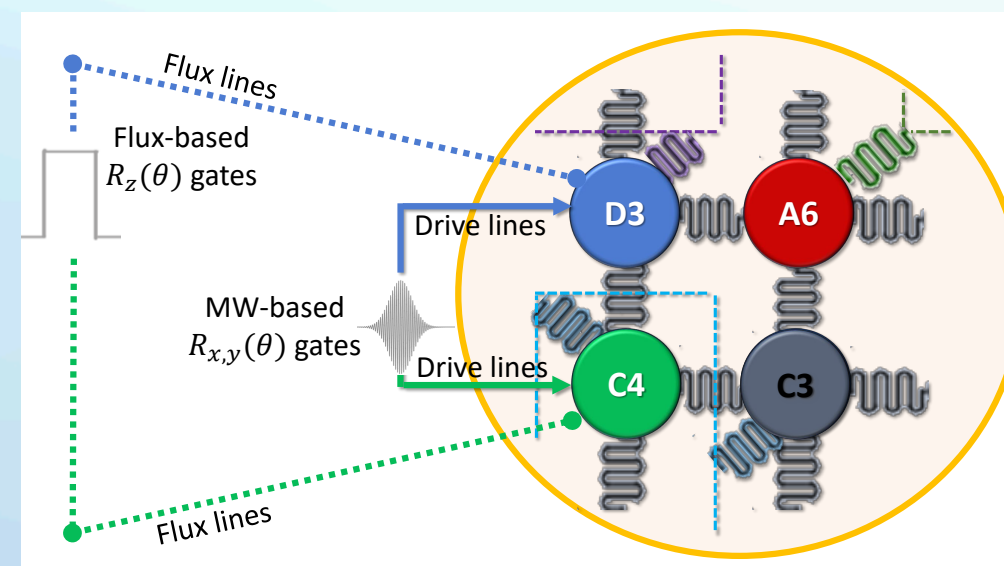
Normalni metal—> kontinualni spektar kvazičestica

Superprovodnik—> otvori 'procep' (gap) u spektru i mogućnost diskretnih nivoa

If one could use the quantum levels in this superconducting gap, these states would be expected to be similar to the electronic state of a free atom, i.e., it is characterized by the long-lived discrete energy spectrum.



Superprovodni kvantni procesor. Shema procesora.



Qubiti (D3-C4) sa kojima smo radili

60+ qubit procesor uskoro dostupan, ali takođe rade na novom tipu qubit-a

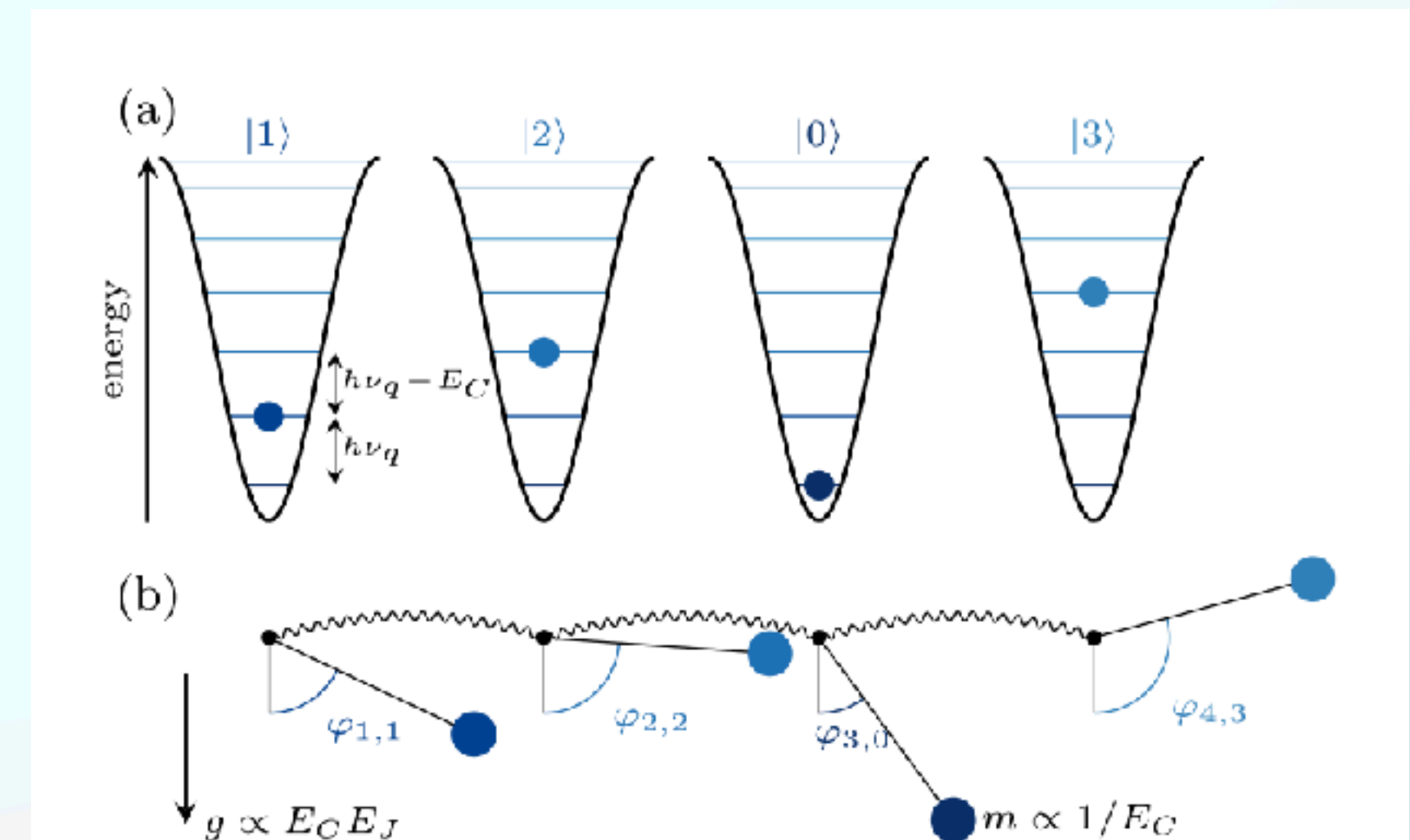



FIG. 1. Quantum vs classical transmon array. (a) A chain of four transmons initialized in the quantum state $|1203\rangle$ of the corresponding cos potentials, where the integers, $i = 1, 2, 0, 3$ correspond to the bound state energies E_i . (b) The corresponding classical rotors, initialized at angular deflections corresponding to the energies E_i as discussed in the main text, and the springs connecting the suspension points representing the angular momentum coupling.

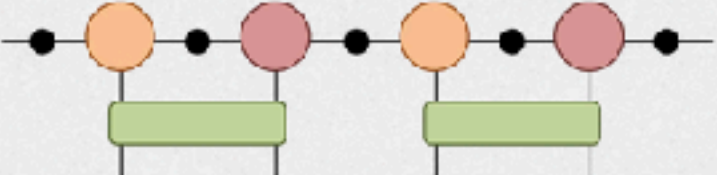
Zašto kvantni računari? —> simulacije mnogočestičnih kvantnih sistemi

- Simulacije veoma izazovne klasično —> Mali broj konstituenata ili aproksimacije za termodinamički relevantne i velike sisteme
- Uvid iz polja kondenzovane materije: efikasne simulacije koristeći tenzorke metode —> Matrix Product States and Density Matrix Renormalization Group: David Pérez-García, Norbert Schuch, and Frank Verstraete, Rev. Mod. Phys. **93**, 045003 (2021)

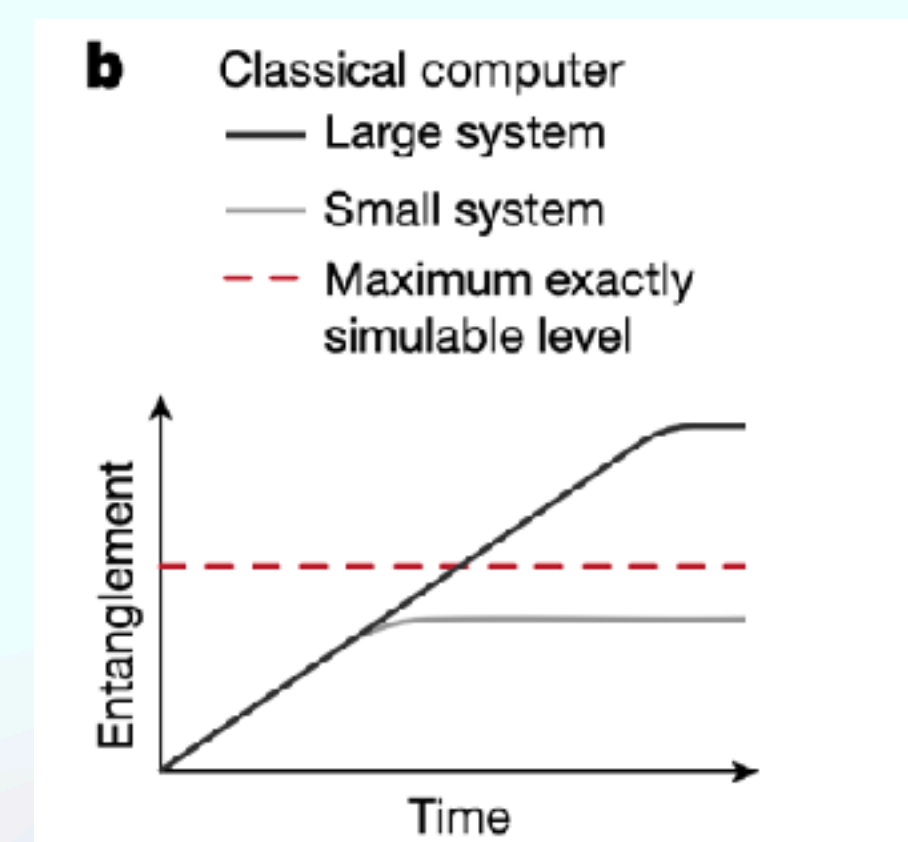
Density matrix renormalization group (DMRG):
widely used and powerful method for the study of quantum systems on $D=1$ dimensional lattices and small-width $D=2$ dimensional lattices. Our code implements DMRG for finite systems with open boundary conditions, following closely the **original implementation** of White. Two versions are presented: one for **matrix product operator** (MPO) Hamiltonians and the other for local Hamiltonians written as a sum of two-body terms.



Time evolving block decimation (TEBD) for matrix product states (MPS):
method for implementing real or imaginary time evolution of matrix product states (MPS), based on a **suzuki-trotter decomposition** of a local Hamiltonian. Our code, which follows closely the **original proposal** of Vidal, demonstrates the use of imaginary time evolution to find the ground state of an infinite (translation-invariant) quantum system in $D=1$ dimensions.



tensors.net



Shaw, A.L., Chen, Z., Choi, J. *et al.* Benchmarking highly entangled states on a 60-atom analogue quantum simulator. *Nature* **628**, 71–77 (2024). <https://doi.org/10.1038/s41586-024-07173-x>

Zapravo, kvantni računari bi trebalo da pomognu baš pri efikasnoj simulaciji kvantnih sistema!

Kvantno-informatički uvid

- Specifični tip kvantnih kola
- Isključivo kvantna kola sa **Clifford** elementima gde su završna stanja veoma **kvantno spregnuta (zapletenost ili 'entanglement')** stabilizujuća kola

Hadamard $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Fazni element $S = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\pi/2} \end{bmatrix}$

kontrolisano+NOT $CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

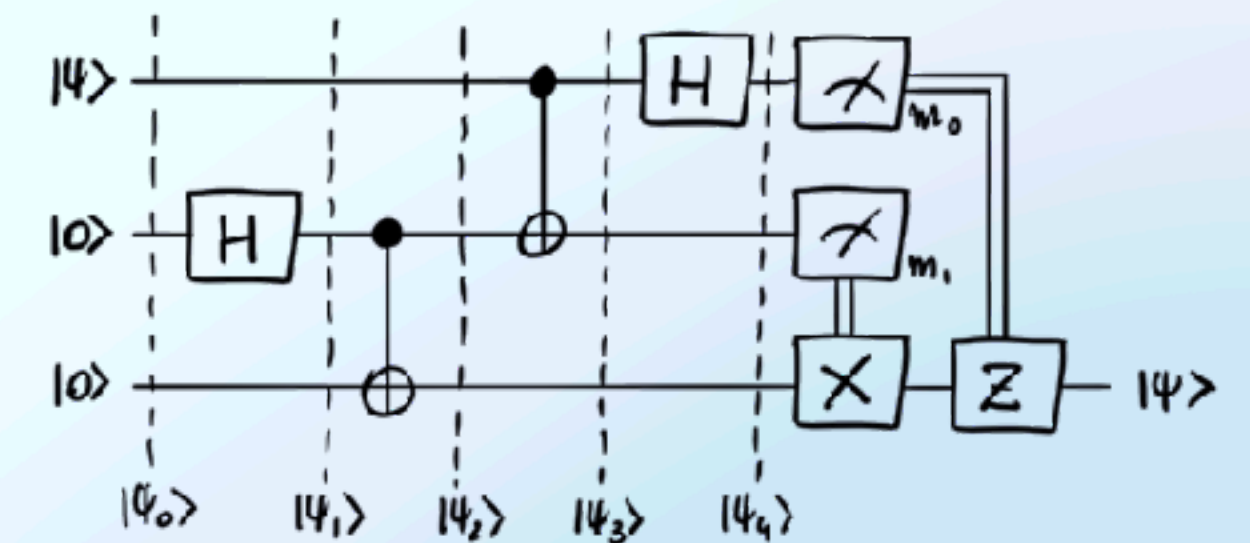
- Ne može svako kvantno kolo da se predstavi Clifford elementima!

Diskretna kvantna mehanika

Svaka validna formulacija/representacija kvantne mehanike sačinjena je od 3 osnovna elementa

1. Priprema stanja
2. Unitarna evolucija → **Kvante kapije (quantum gates)**
3. Merenje

Protokol za kvantnu teleportaciju kvantnog stanja



Comptine d'un autre été
 Claivert's Piano Arrangement
 Music by Yann Tiersen

♩ = 100

Muzičke note/ sa leva na desno. Početak, note i kraj.

*aproksimacija kontinuirane dinamike → Troterizacija

Opovrgavanje Guglove tvrdnje o kvantnoj nadmoć iz 2019

Jedan od pristupa je koristio blago perturbovana Clifford kola

Article | Published: 23 October 2019

Quantum supremacy using a programmable superconducting processor

[Frank Arute](#), [Kunal Arya](#), [Ryan Babbush](#), [Dave Bacon](#), [Joseph C. Bardin](#), [Rami Barends](#), [Rupak Biswas](#), [Sergio Boixo](#), [Fernando G. S. L. Brandao](#), [David A. Buell](#), [Brian Burkett](#), [Yu Chen](#), [Zijun Chen](#), [Ben Chiaro](#), [Roberto Collins](#), [William Courtney](#), [Andrew Dunsworth](#), [Edward Farhi](#), [Brooks Foxen](#), [Austin Fowler](#), [Craig Gidney](#), [Marissa Giustina](#), [Rob Graff](#), [Keith Guerin](#), ... [John M. Martinis](#)

Show authors

[Nature](#) **574**, 505–510 (2019) | [Cite this article](#)



Dobitnik Nobelove nagrade

N.B. Preskoćiću celokupno polje **'kvatne korekcije grešaka' (quantum error correction)** gde Clifford operacije igraju veoma bitnu ulogu → sledeća Nobelova nagrada!

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RESEARCH ARTICLE | PHYSICS

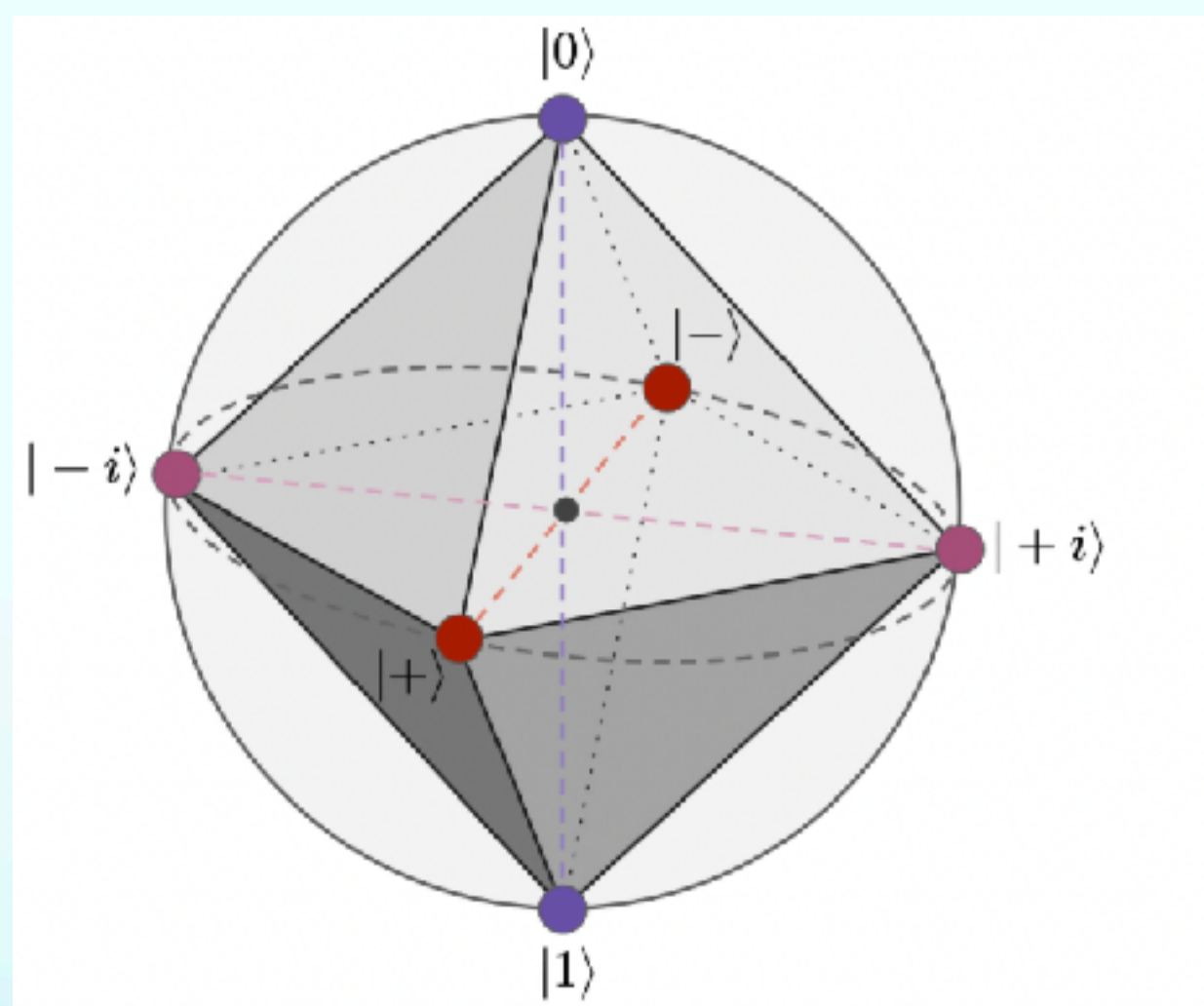
Fast and converged classical simulations of evidence for the utility of quantum computing before fault tolerance

TOMISLAV BEGUŠIĆ, JOHNIE GRAY, AND GARNET KIN-LIC CHAN

SCIENCE ADVANCES • 17 Jan 2024 • Vol 10, Issue 3 • DOI: 10.1126/sciadv.adk4321

Stabilizatori stanja na primeru jednog qubit-a

ograničen Hilbertov prostor → 6 čistih kvantnih stanja

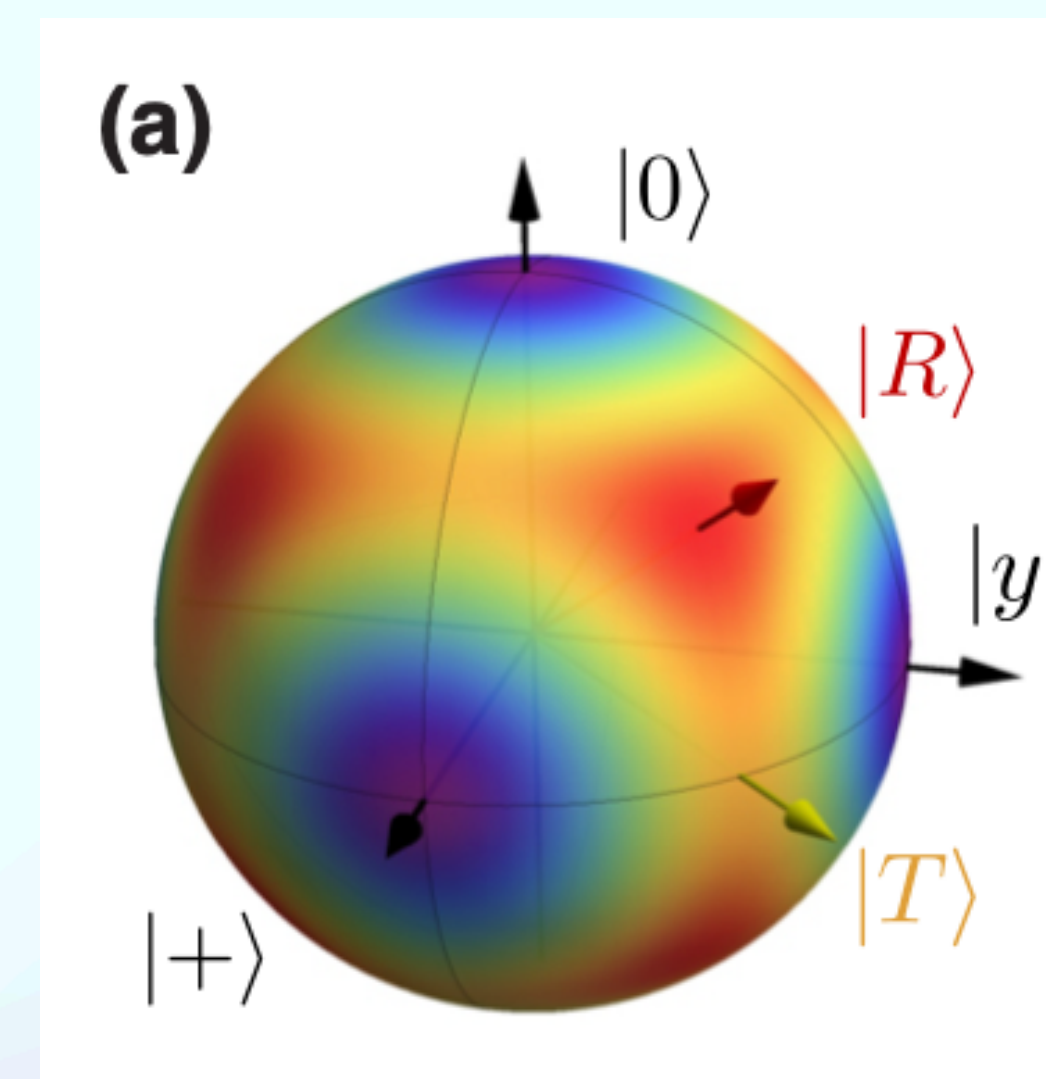


Oktaedar na Blohovoju sferi →
Koristeći samo 1-qubit kvantne
kapije (https://pennylane.ai/qml/demos/tutorial_clifford_circuit_simulations).

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\pi/2} \end{bmatrix}$$

Ne-Clifford kvantne kapije omogućavaju istraživanje celokupnog Hilbertovog prostora



PRX QUANTUM **4**, 010301 (2023)

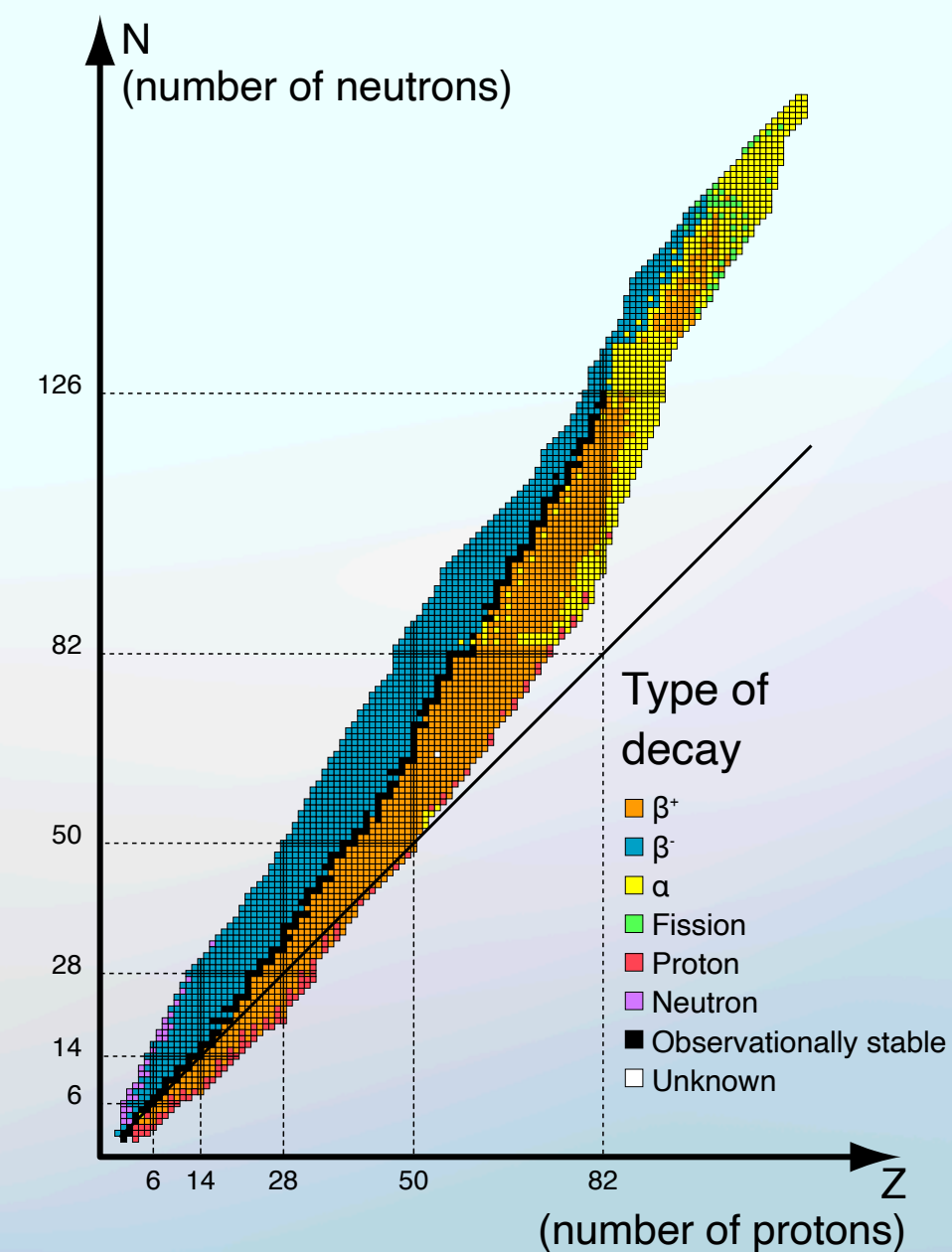


Sve van ovih 6 stanja → **ne-stabilizatornost ili "magija"**

Magija u matematici i fizici?

- Magični kvadrati, magični brojevi
- Magični broj nukleona u jezgru koji ga čini stabilnim (2, 8, 20, 28, 50, 82, 126)
- Magični ugao u grafenu

2	7	6	→15	
9	5	1	→15	
4	3	8	→15	
↙15	↓15	↓15	↓15	↘15



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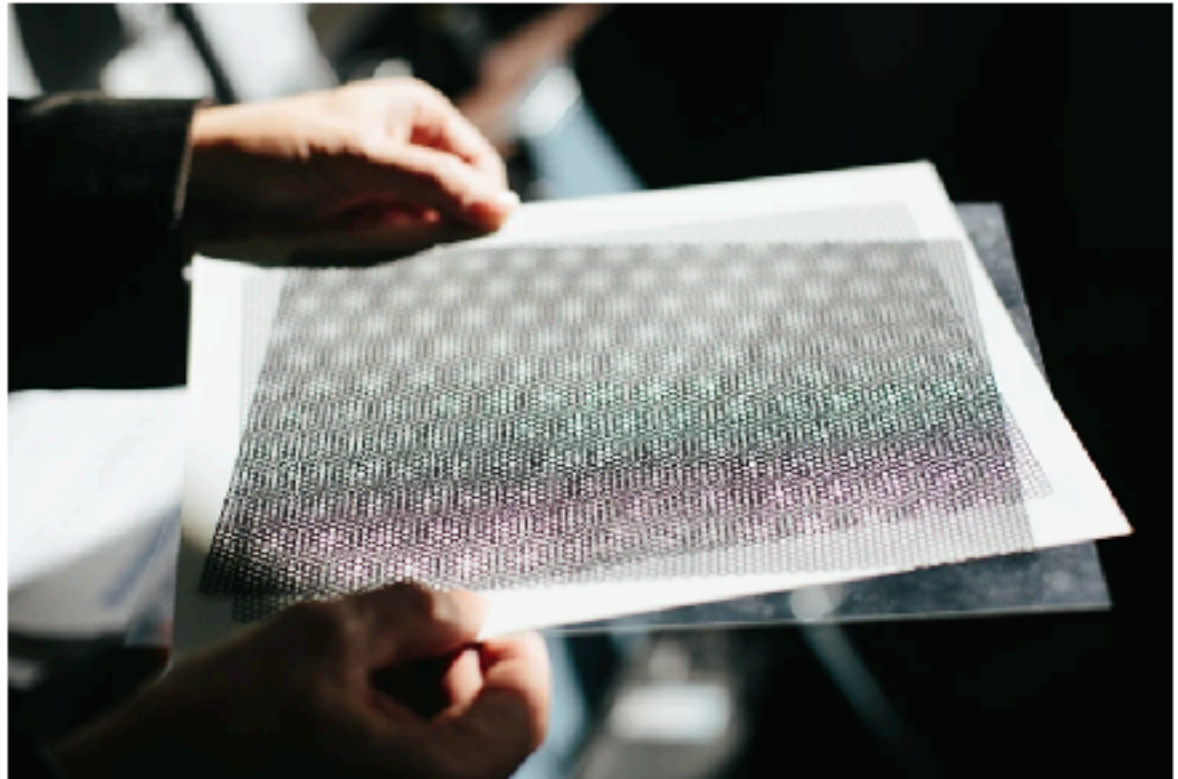
NEWS FEATURE | 02 January 2019

How 'magic angle' graphene is stirring up physics

Misaligned stacks of the wonder material exhibit superconductivity and other curious properties.

By [Elizabeth Gibney](#)

[Email](#) [Twitter](#) [Facebook](#) [LinkedIn](#) [Apple](#) [WhatsApp](#) [X](#)



Overlapping two sheets of graphene shows a characteristic pattern. Credit: Juliette Halsey for Nature

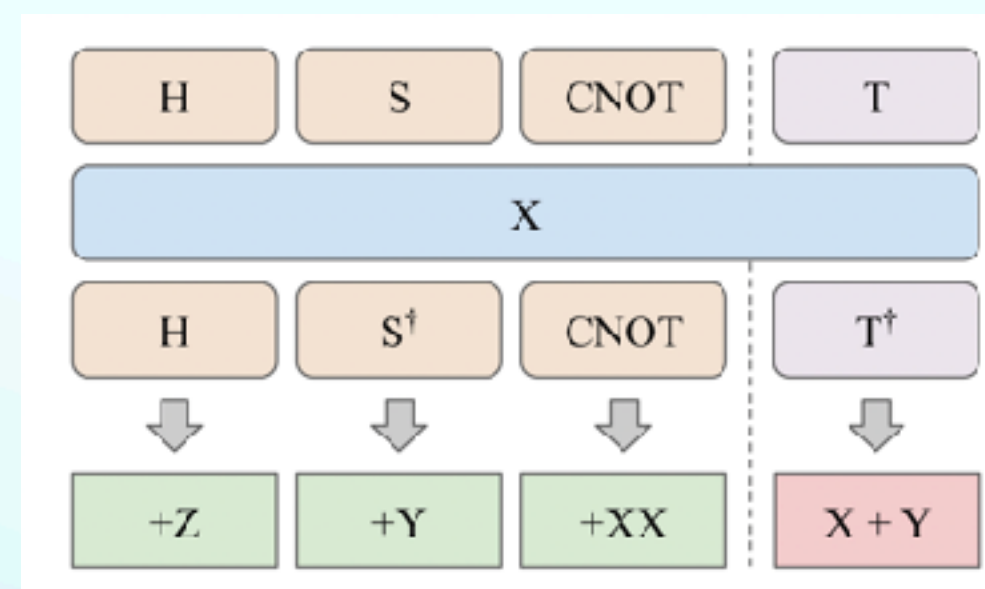
Paradigma Clifford+T kvantnih kola

- Clifford metod za simulaciju kvantnih kola ne funkcioniše ukoliko se dodaju **T elementi** (1-qubit)

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

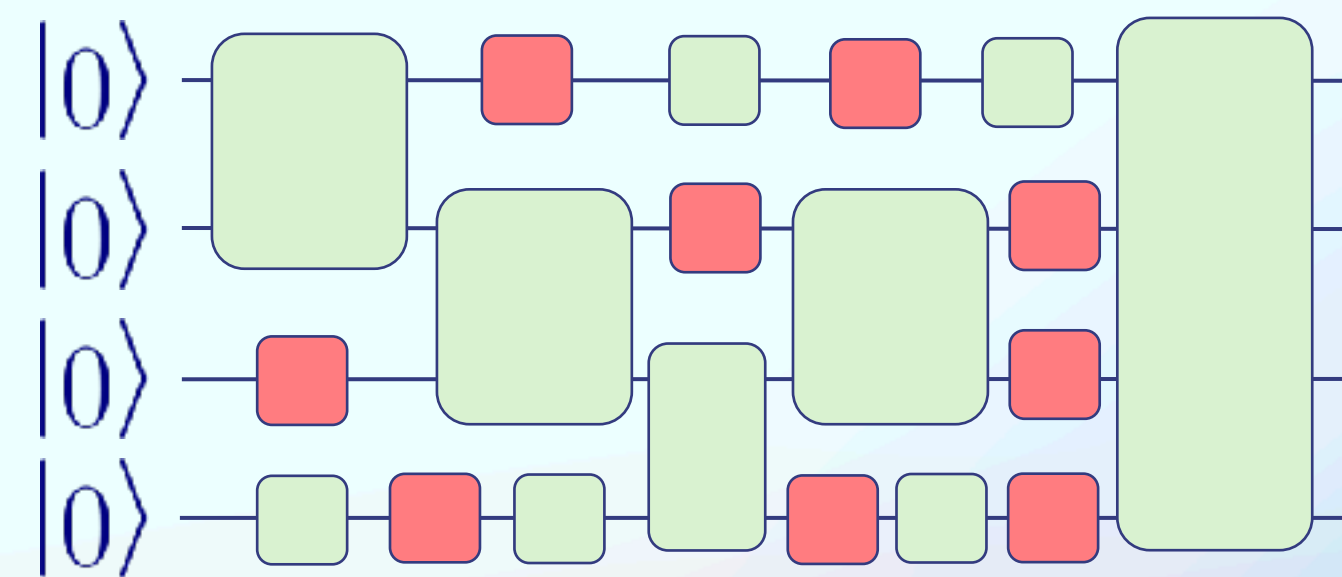
- Eksponencijalno teža simulacija ukoliko je magija prisutna → Aaronson, S. and Gottesman, D. 'Improved Simulation of Stabilizer Circuits', Physical Review A, **70**(5) (2004)

Dinamika operatora



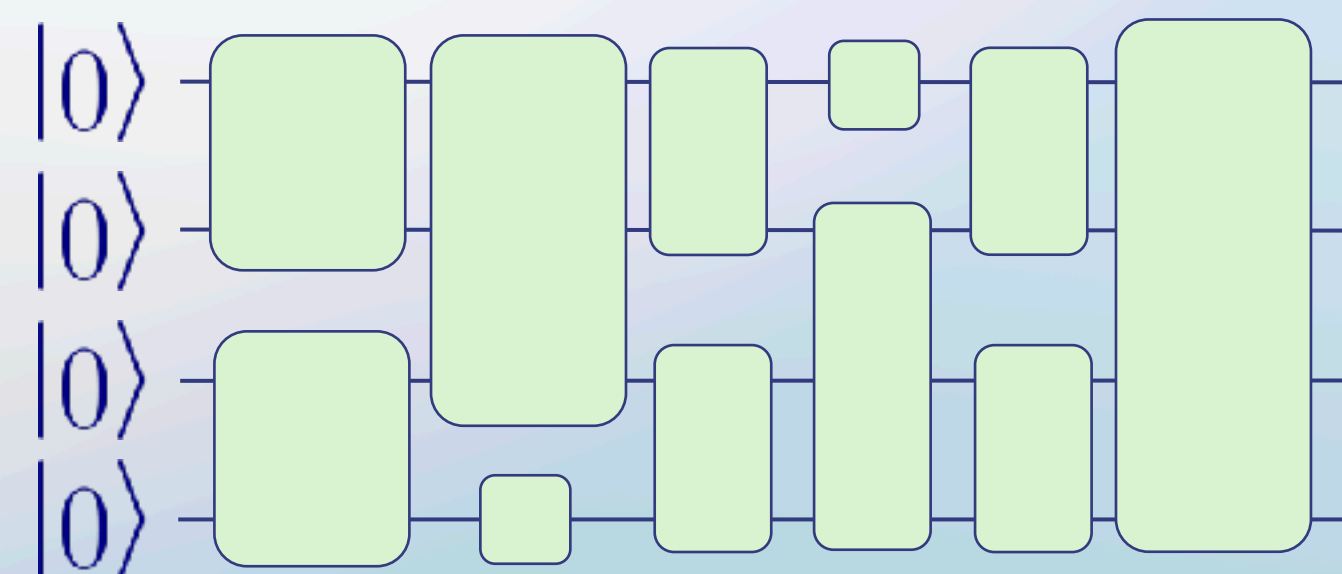
(https://pennylane.ai/qml/demos/tutorial_clifford_circuit_simulations).

cena simulacije kvantnih kola



$\exp(n)$

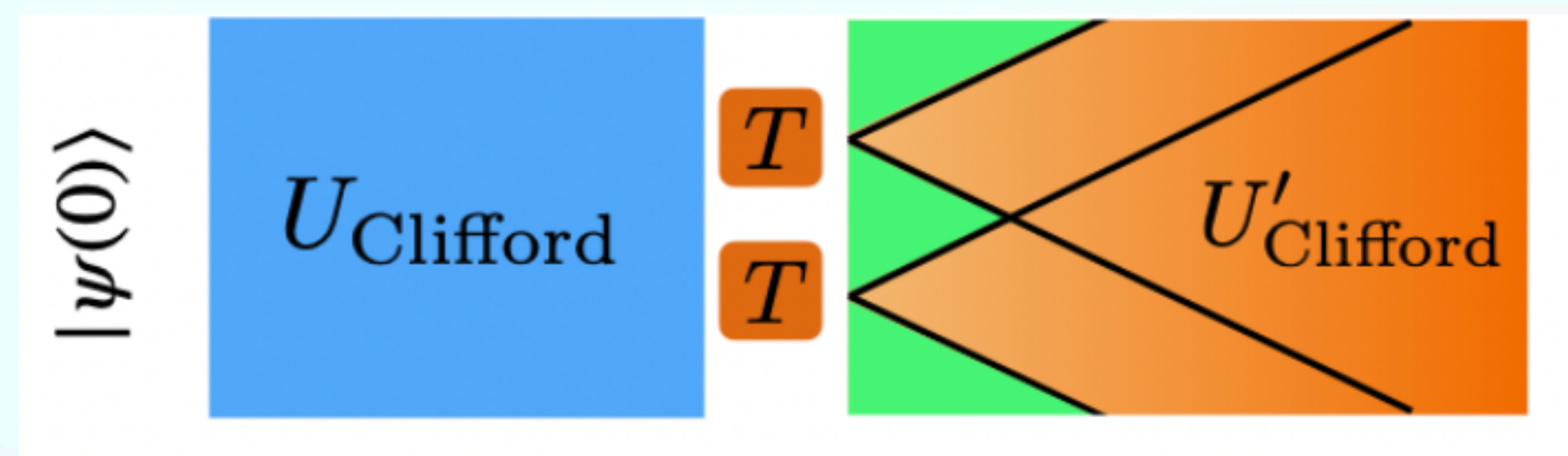
Magija



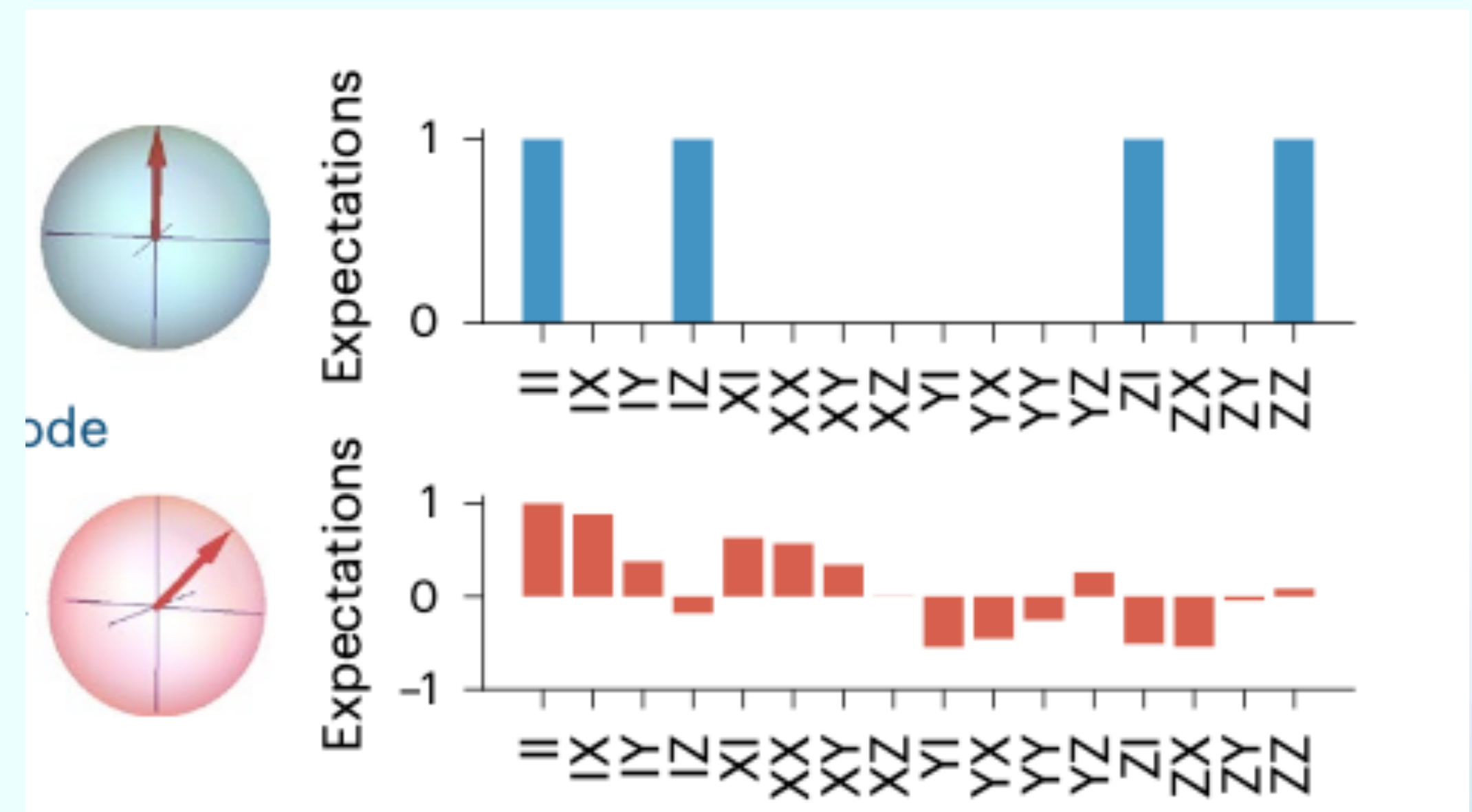
$\text{poly}(n)$

bez magije

Efekti T elemenata



SciPost Phys. 9, 087 (2020)



Niroula, P., White, C.D., Wang, Q. *et al.* Phase transition in magic with random quantum circuits. *Nat. Phys.* 20, 1786–1792 (2024)

Entropija stabilizacije kao observable kola kvantifikuje 'udaljenost' od stabilizatora (nula za ta stanja)

L. Leone, S.F.E. Oliviero, A. Hamma PRL **128**, 050402(2022)

$$M_\alpha(\psi) = \frac{1}{1-\alpha} \log_2 \mathcal{P}_\alpha(\psi), \quad \mathcal{P}_\alpha(\psi) = \frac{1}{d} \sum_{P \in \mathcal{P}_N} |\text{Tr}(P\psi)|^{2\alpha}, \quad \psi = |\psi\rangle\langle\psi| \quad d = 2^N$$

Entropija stabilizacije u eksperimentu

... uzorkovanje (sampling) - Nasumična Merenja (Randomized Measurements)

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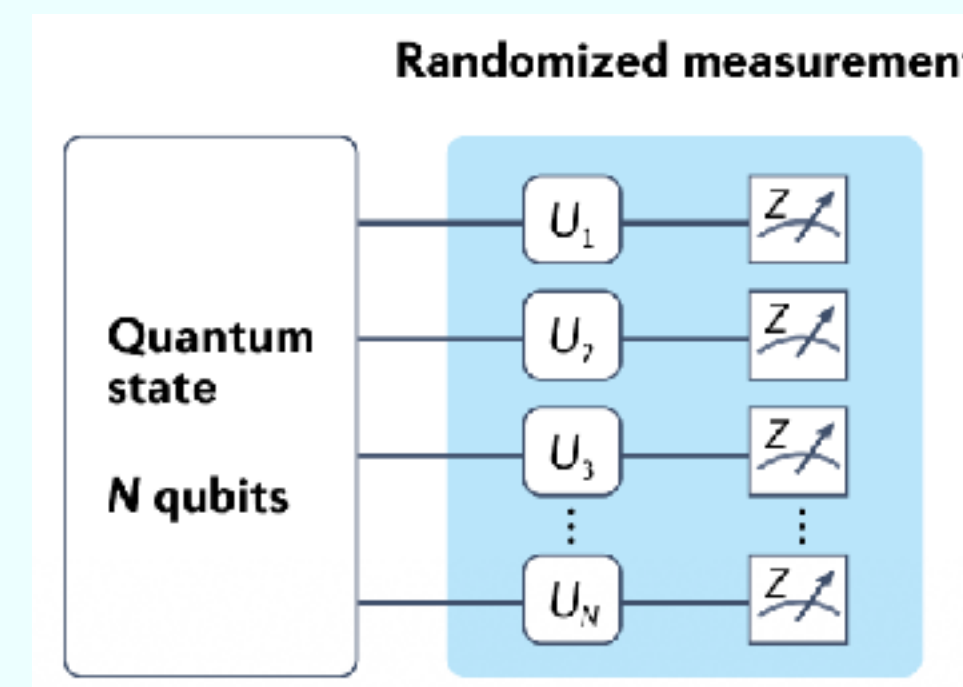
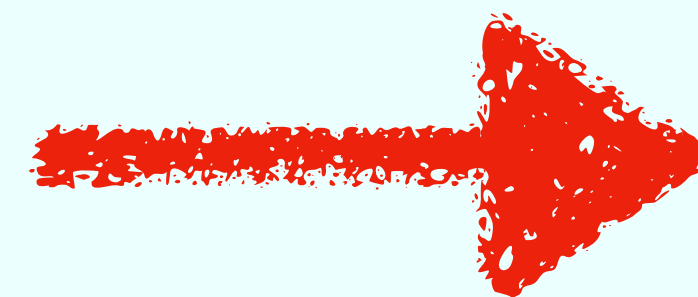
REPORT

Probing Rényi entanglement entropy via randomized measurements

TIFF BRYDGES, ANDREAS ELBEN, PETAR JURCEVIC, BENOÎT VERMERSCH, CHRISTINE MAIER, BEN P. LANYON, PETER ZOLLER, RAINER BLATT, AND CHRISTIAN F. ROOS

Authors Info & Affiliations

SCIENCE • 19 Apr 2019 • Vol 364, Issue 6437 • pp. 260-263 • DOI: 10.1126/science.aau4963



+klasična obrada podataka



slično Monte Karlo metodama u statističkoj fizici

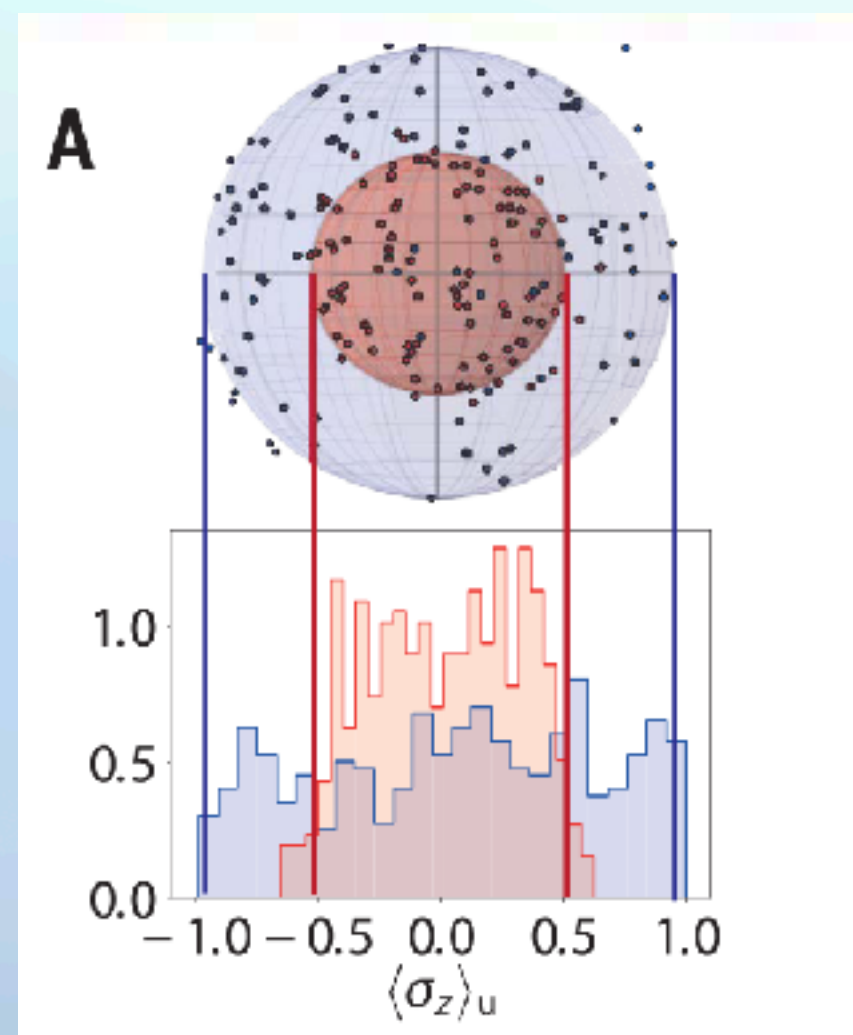
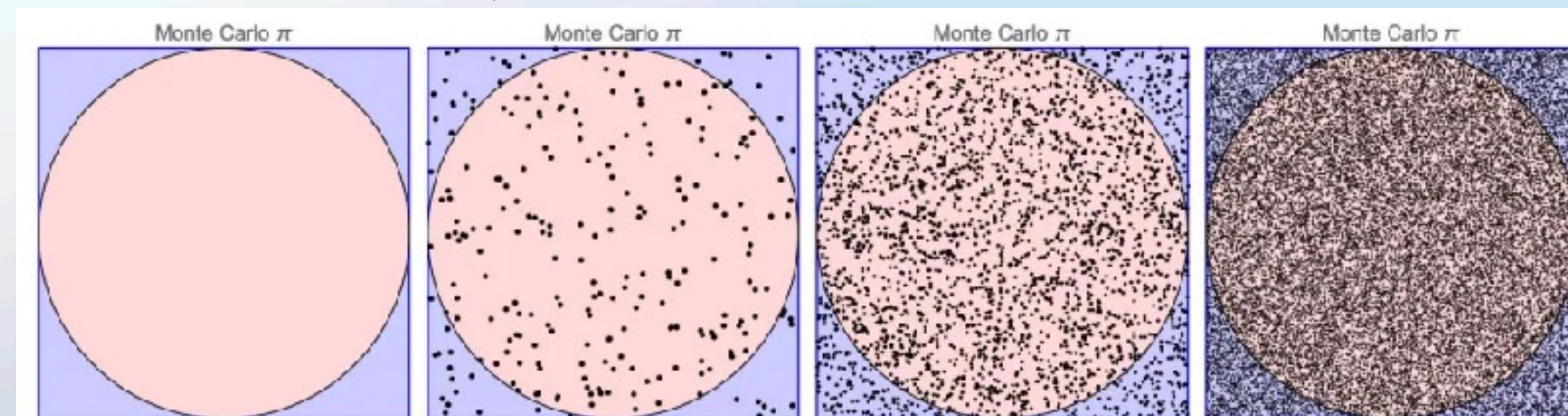


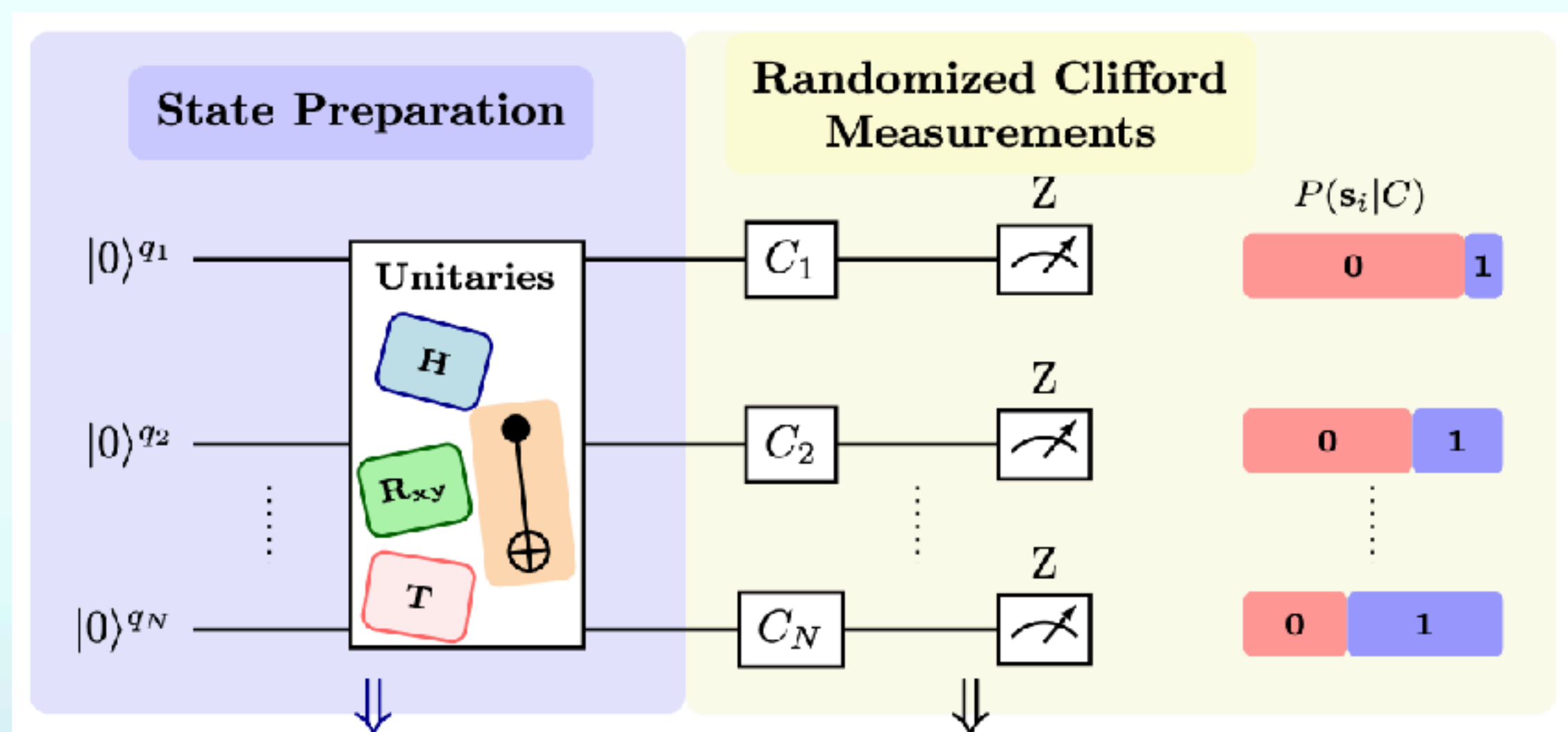
Fig. 1. Measuring second-order Rényi entropies through randomized measurements. (A) Single-qubit Bloch sphere. The purity is directly related to the width of the distribution of measurement outcomes after applying random rotations u_i . Initial pure state (blue) and mixed state (red) cases are shown. See text. (B) Generalization to multiple qubits: Measuring N_A -qubit



Nasumična Clifford Merenja - Randomized Clifford Measurement (RCM)

Oliviero, S.F.E. et al. 'Measuring magic on a quantum processor', npj Quantum Information, 8(1) (2022)

$$M_2(|\psi\rangle) = -\log_2 \mathcal{W} + \log_2 \mathcal{P} - \log_2 d,$$



24 Clifford 1-qubit operators

A/P	I	X	Y	Z
I	I	X	Y	Z
H	H	XH	YH	ZH
HS	HS	XHS	YHS	ZHS
S	S	XS	YS	ZS
SH	SH	XSH	YSH	ZSH
HSH	HSH	XHSH	YHSH	ZHSH

Table I: Single-qubit Clifford gates (modulo global phase $U(1)$) used in Eqs. (2) and (3), where the columns are Pauli operators $P = \{I, X, Y, Z\}$ and rows are coset representatives $A = \{I, H, HS, S, SH, HSH\}$. H is the Hadamard gate, and S is the $\pi/2$ phase gate.

RCM protocol → uzorkovanje stanja sa identičnom količinom magije

$$\mathcal{P} = d \sum_{\vec{s}} (-2)^{-\|\vec{s}\|_2} \mathbb{E}_C [P(\mathbf{s}_1|C)P(\mathbf{s}_2|C)], \quad \mathcal{W} = - \sum_{\vec{s}} (-2)^{-\|\vec{s}\|_4} \mathbb{E}_C [P(\mathbf{s}_1|C)P(\mathbf{s}_2|C)P(\mathbf{s}_3|C)P(\mathbf{s}_4|C)].$$

Šta je nelokalna magija?

- Nelokalna magija postoji u stanju koje je kvantno spregnuto (entangled): arXiv:2403.07056
- Protokol: pripremiti stanja sa nelokalnom magijom i zatim pokušati izbrisati lokalni deo

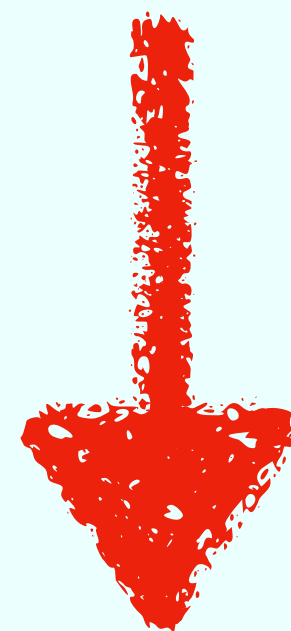
$$M_2^{\text{NL}}(|\psi\rangle) = \min_{U_A, U_B} M_2(U_A \otimes U_B |\psi\rangle)$$



Vizuelizacija kvantne spletenosti i nelokalne magije u slučaju dva qubit-a ili kvantna spina-1/2. (Ilustracija: prof. dr. Slobodan Radošević)



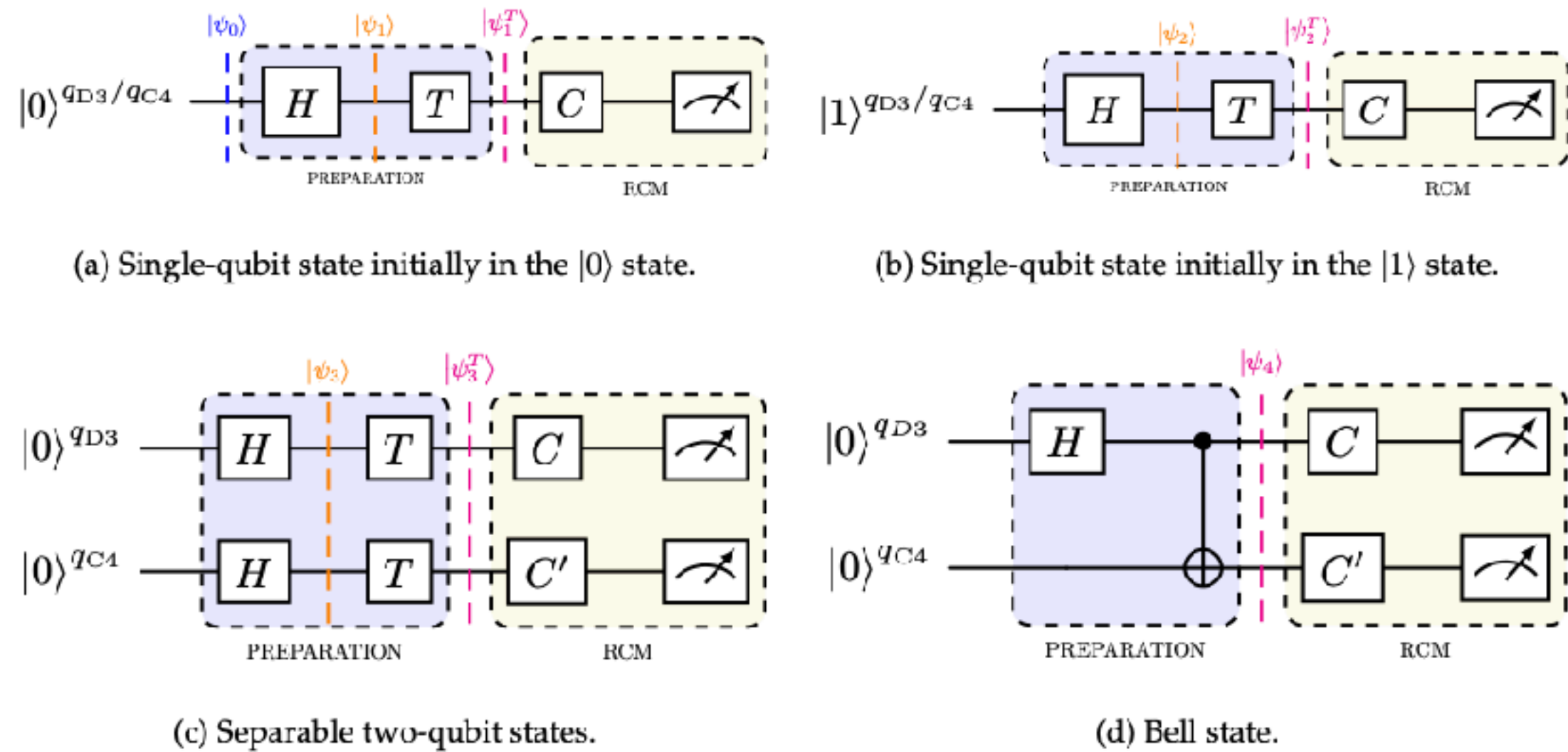
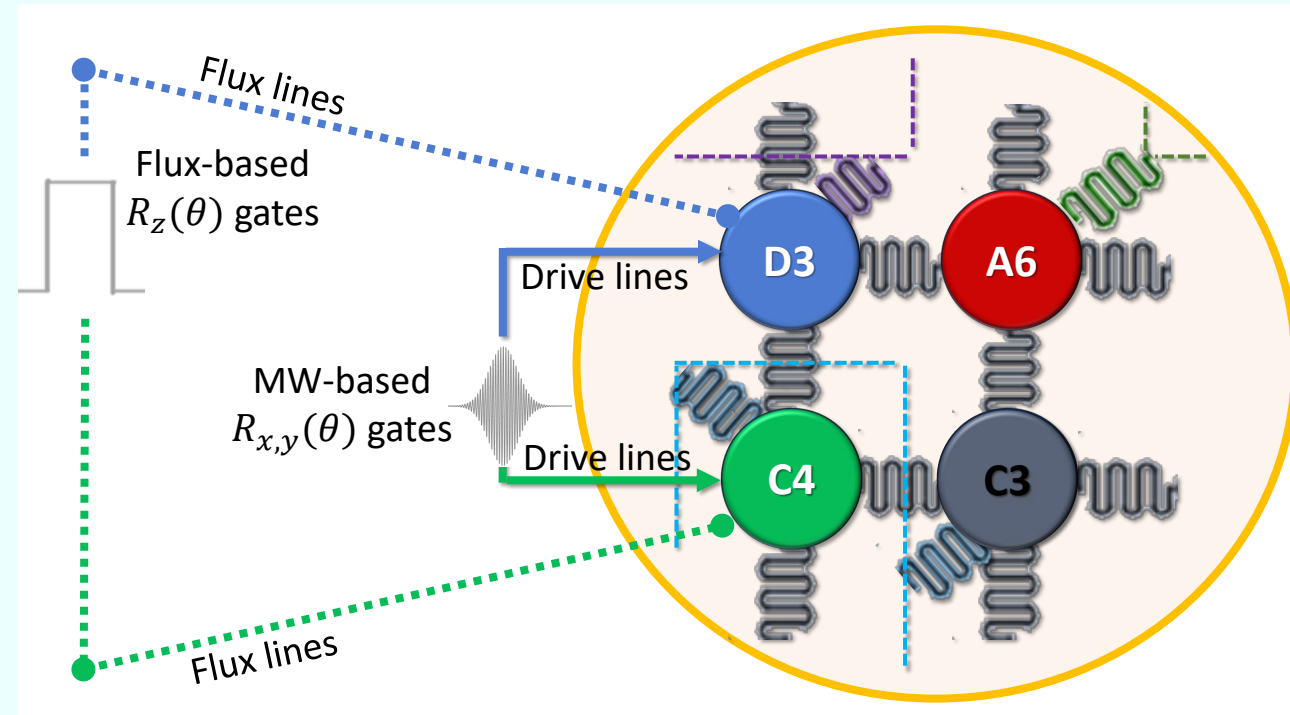
Nelokalna magija još nije eksperimentalno detektovana



Stoga smo je po prvi put izmerili na kvantnom procesoru!

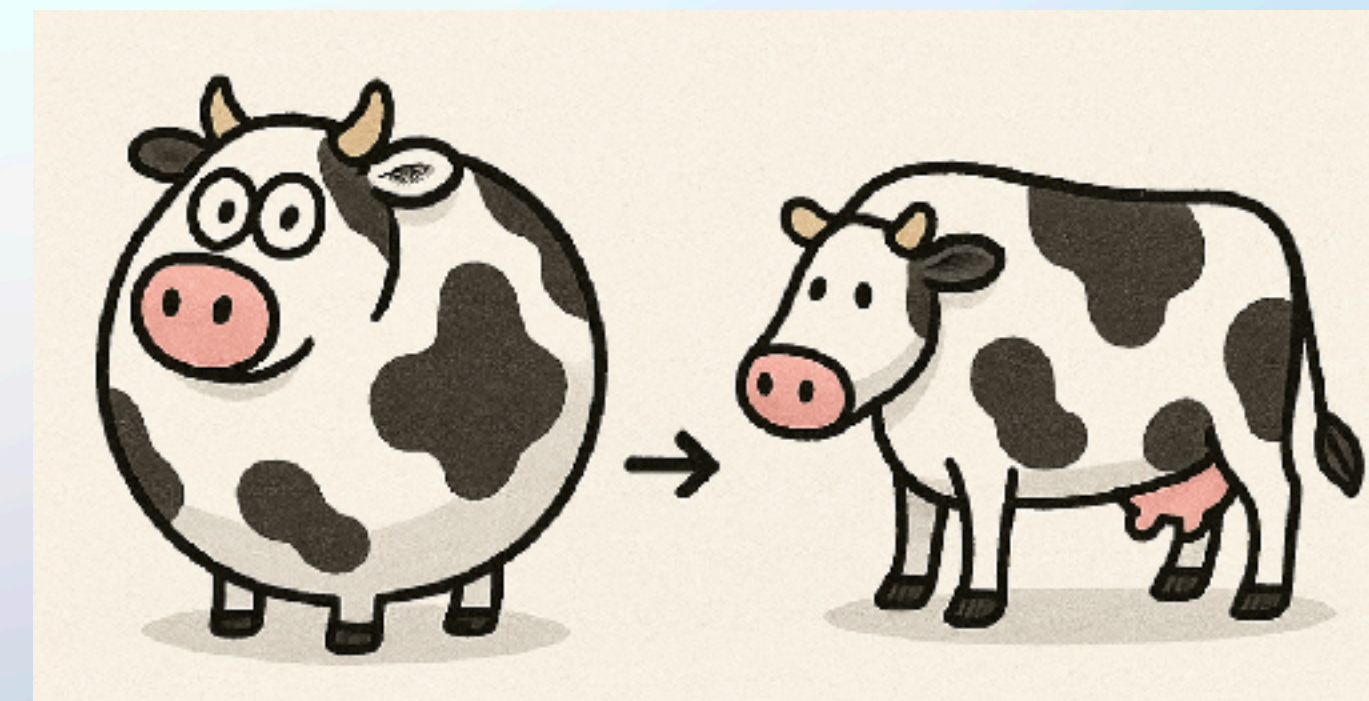
	Class 1	Class 2	Class 3
Notation	Local Magic(LM)	Local+Non-Local Magic(M)	Non-Local Magic(NLM)
Local	✓	✓	✗
Non-Local	✗	✓	✓
Local Erasure	Complete	Partial	Null

Referentni rezultati (benchmark) i identifikacija izvora šuma (noise)



	$F_{RO}(\%)$	$F_{RB}(\%)$	$T_2^{Echo}(\mu s)$	$T_2^*(\mu s)$	$T_1(\mu s)$	$M_2^{exp}(\Psi^{M_2^h=0})$	$M_2^{exp}(\Psi^{M_2^h \neq 0})$
(a)							
D3	94 ± 1	99.75 ± 0.01	31.1 ± 0.7	25 ± 6	23.3 ± 0.1	$ \Psi_0\rangle$	0.1 ± 0.1
	96 ± 1	99.910 ± 0.002	39.9 ± 0.7	33 ± 6	30.0 ± 0.1	$ \Psi_1\rangle$	0.5 ± 0.1
C4	95 ± 1	99.925 ± 0.003	38.8 ± 1.1	24 ± 2	36.2 ± 0.1	$ \Psi_1\rangle$	0.4 ± 0.1
						$ \Psi_2\rangle$	0.4 ± 0.1
(b)							
	$F_{RO}(\%)$	$F_{RB}(\%)$	$P_{RB}^{ZZ}(\%)$	$Cross^{MW}(\%)$	$Cross^{ZZ}(kHz)$		
D3	83 ± 1	99.746 ± 0.014	99 ± 2	~ 0.12	~ 99	$ \Psi_3\rangle$	0.8 ± 0.1
C4	91 ± 1	99.905 ± 0.003		~ 1.57	~ 100	$ \Psi_3\rangle$	0.8 ± 0.1
(c)							
D3	92 ± 1					$ \Psi_4\rangle$	0.2 ± 0.1
C4							

Table 1: Experimental magic measured for the quantum states in Fig. 3. Systematic investigation of the experimental magic for single- and two-qubit quantum states, involving qubits D3 and C4, as a function of: the readout fidelity F_{RO} , the average single-qubit gates fidelity measured by Randomized Benchmarking, F_{RB} , both in the isolated case (a) and simultaneously (b), statistical relaxation, Ramsey and Echo coherence times (T_1 , T_2^* and T_2^{Echo} , respectively). For the two-qubit circuits, we have also reported an estimation of the drive and ZZ crosstalk on the investigated pair [55–58]. A discussion on the error estimation is reported in App. B.



Naš posao kao teoretičara

1.2 Strategies to counter errors

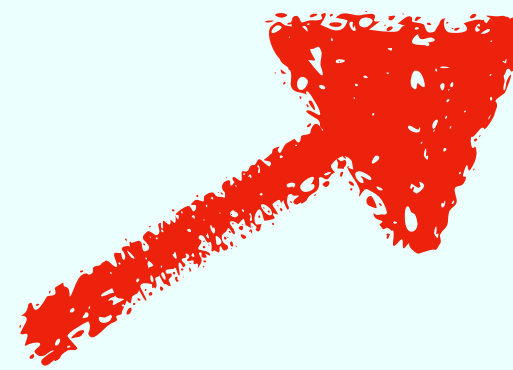
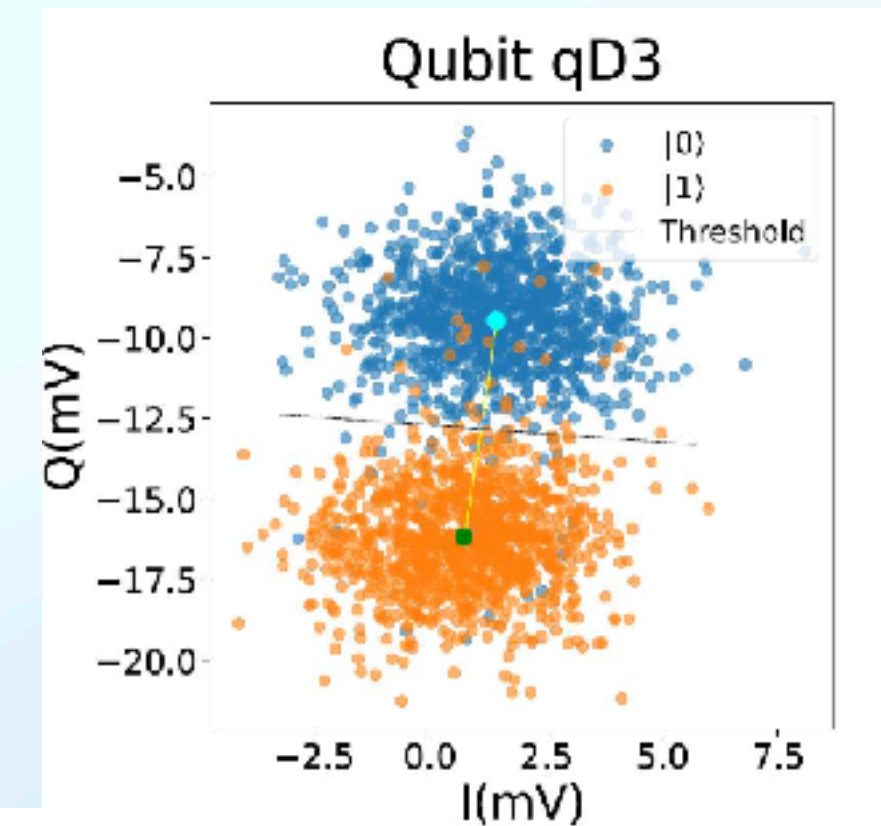
1. **Do nothing and hope for the best.** We can only run short circuits before our computation is overwhelmed by the noise. This regime is called the *Noisy Intermediate-Scale Quantum (NISQ) regime* [16].

Error Mitigation and Error Correction Lecture Notes - Philippe Faist

...je bio da razumemo i razložimo izvore grešaka koje naš procesor ima na osnovu preliminarnih i benchmark eksperimenata

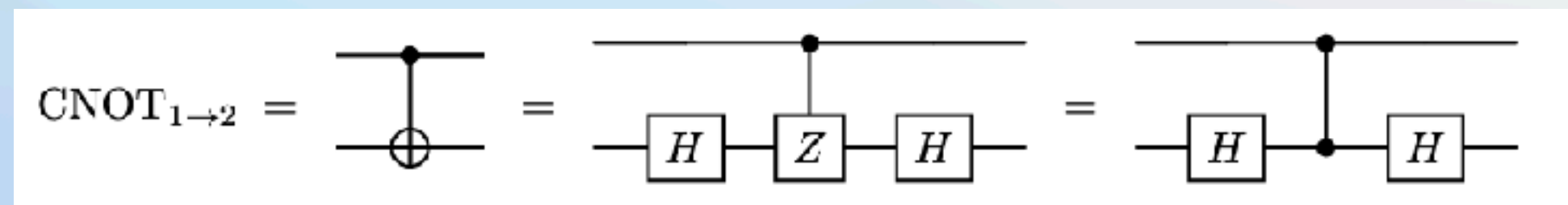
Nasumični bit-flip

$0 \rightarrow 1$ or $1 \rightarrow 0$.



1. Greške pri samom merenju

2. CNOT kvantni elementi (depolarizujući šum)



$$\mathcal{E}(\rho) = \frac{pI}{2} + (1-p)\rho. \quad (8.100)$$

The effect of the depolarizing channel on the Bloch sphere is illustrated in Figure 8.11.

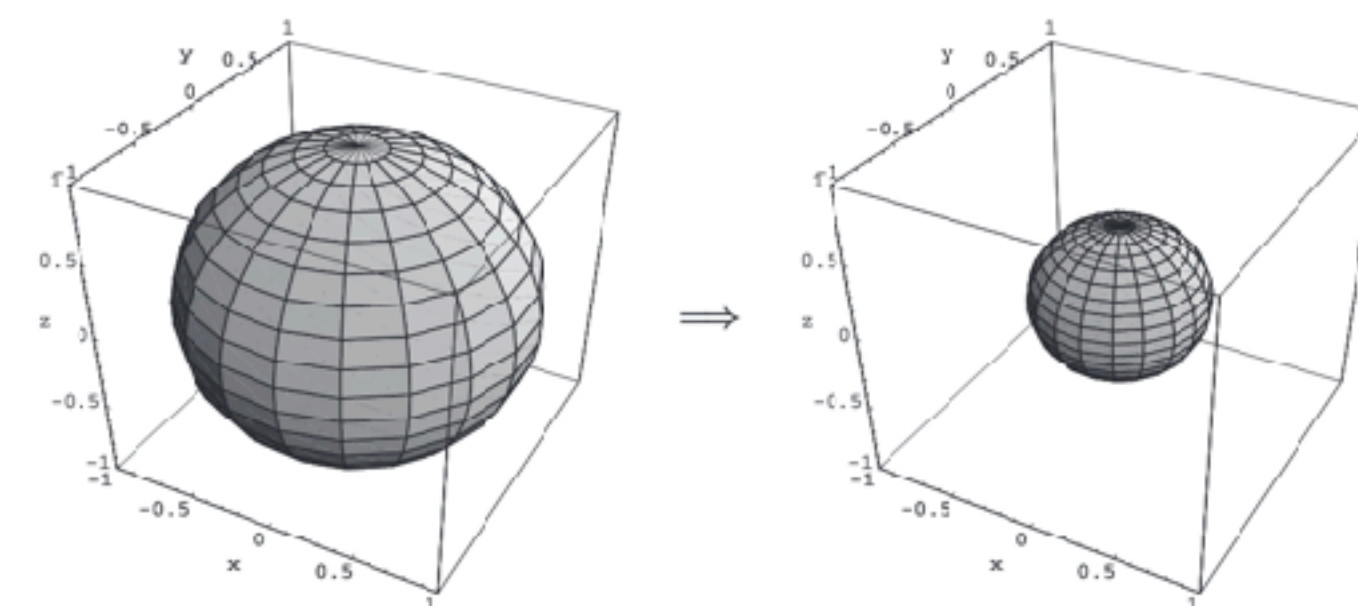
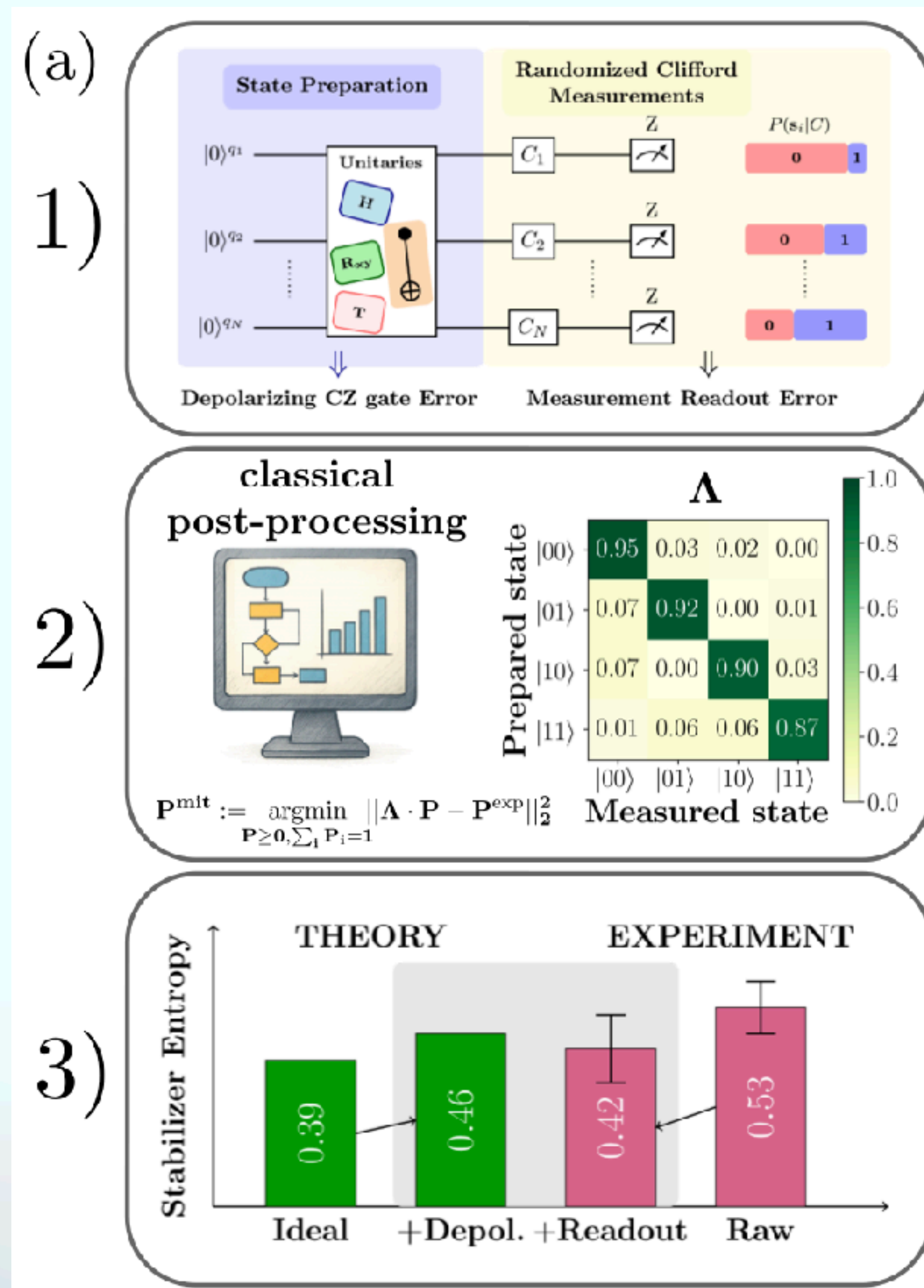


Figure 8.11. The effect of the depolarizing channel on the Bloch sphere, for $p = 0.5$. Note how the entire sphere contracts uniformly as a function of p .

Workflow

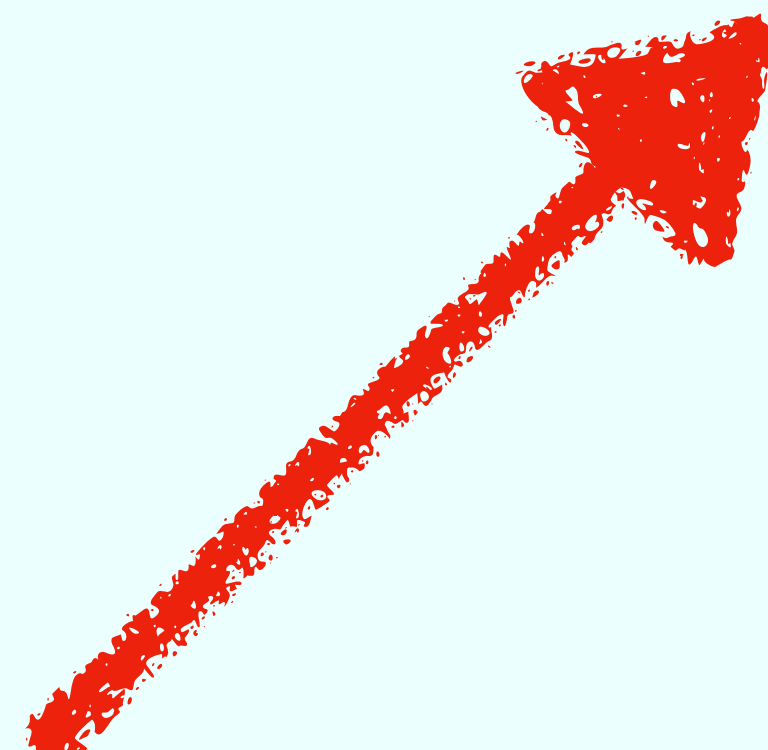
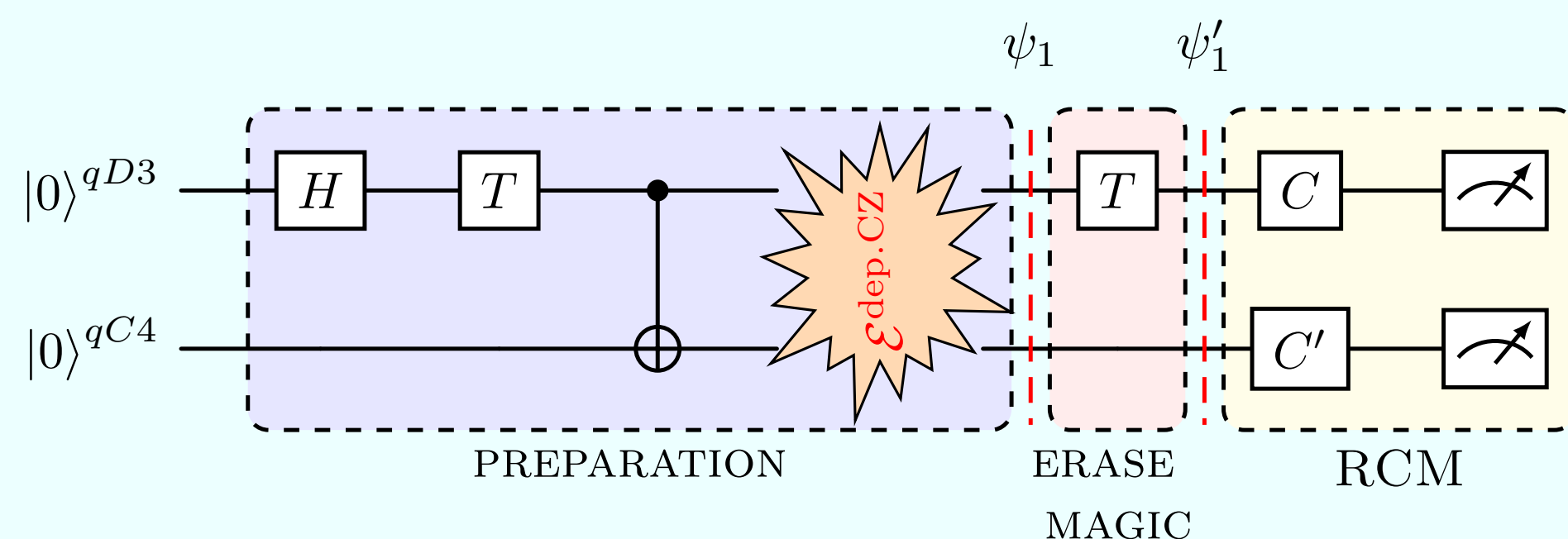
priča o sjedinjavanju teorije i eksperimenta



dvokraki pristup

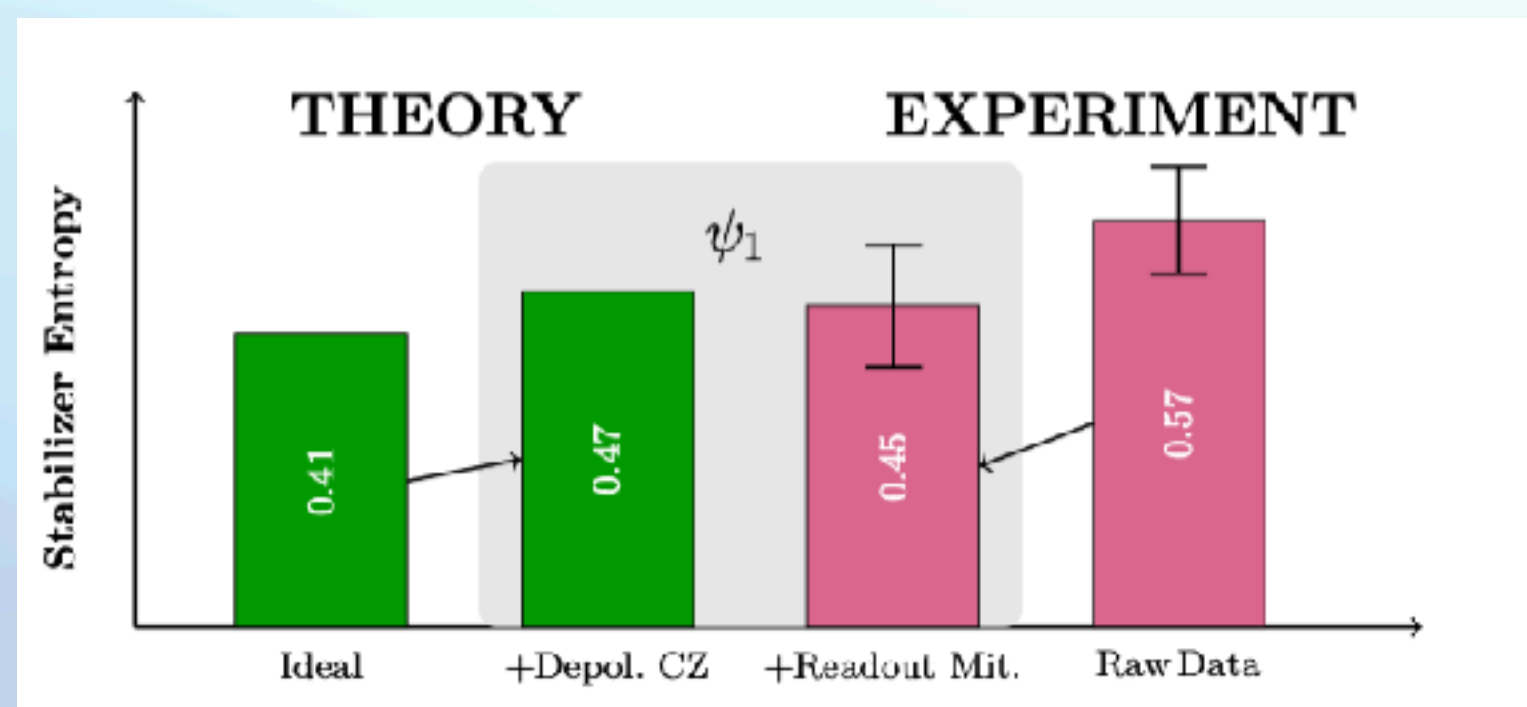
Klasa 1: Lokalna Magija

Alternativni metod



Circuit	State	Th. Purity	Exp. Purity	Th. Magic	Exp. Magic	NL-Magic (from RDM purity)
Preparation	ψ_1	0.92	1.02 ± 0.04	0.47	0.45 ± 0.06	
Prep.+Erasure	ψ'_1	0.92	1.05 ± 0.06	0.11	0.08 ± 0.08	0.1176 ± 0.0007

Table 2: Local Magic (LM) state results. Comparison of Purity and Magic for states ψ_1 (before) and ψ'_1 (after) the erasure, as well as the value of non-local magic obtained from subsystem purity. The theoretical predictions for purity and magic are calculated by considering the CZ depolarizing error and compared with the experimental results after readout error mitigation. Details on the errors are reported in *Methods*.



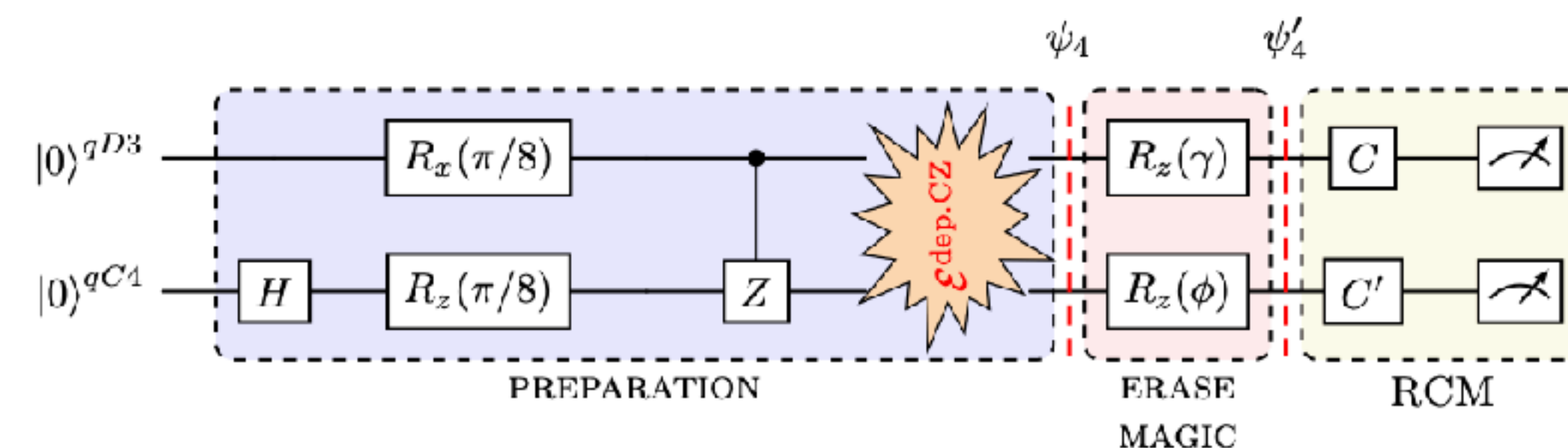
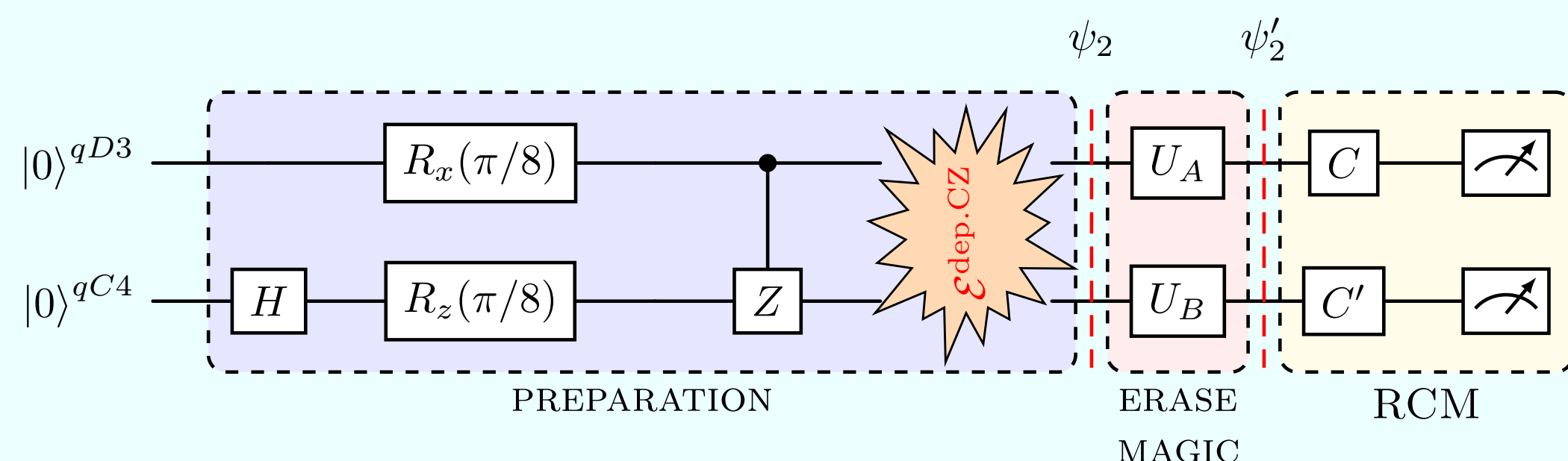
$$|\psi\rangle = \sqrt{\lambda}|\psi_1^A\rangle|\psi_1^B\rangle + \sqrt{1-\lambda}|\psi_2^A\rangle|\psi_2^B\rangle$$

$$\psi_A := \text{Tr}_B(|\psi\rangle\langle\psi|) \quad \text{Pur}(\psi_A) = \lambda^2 + (1-\lambda)^2.$$

$$M^{\text{NL}}(|\psi\rangle) = -\log_2(4\text{Pur}(\psi_A)^2 - 6\text{Pur}(\psi_A) + 3).$$

Closed-form za slučaj 2 qubit-a

Klasa 2: Lokalna+Nelokalna magija (M)

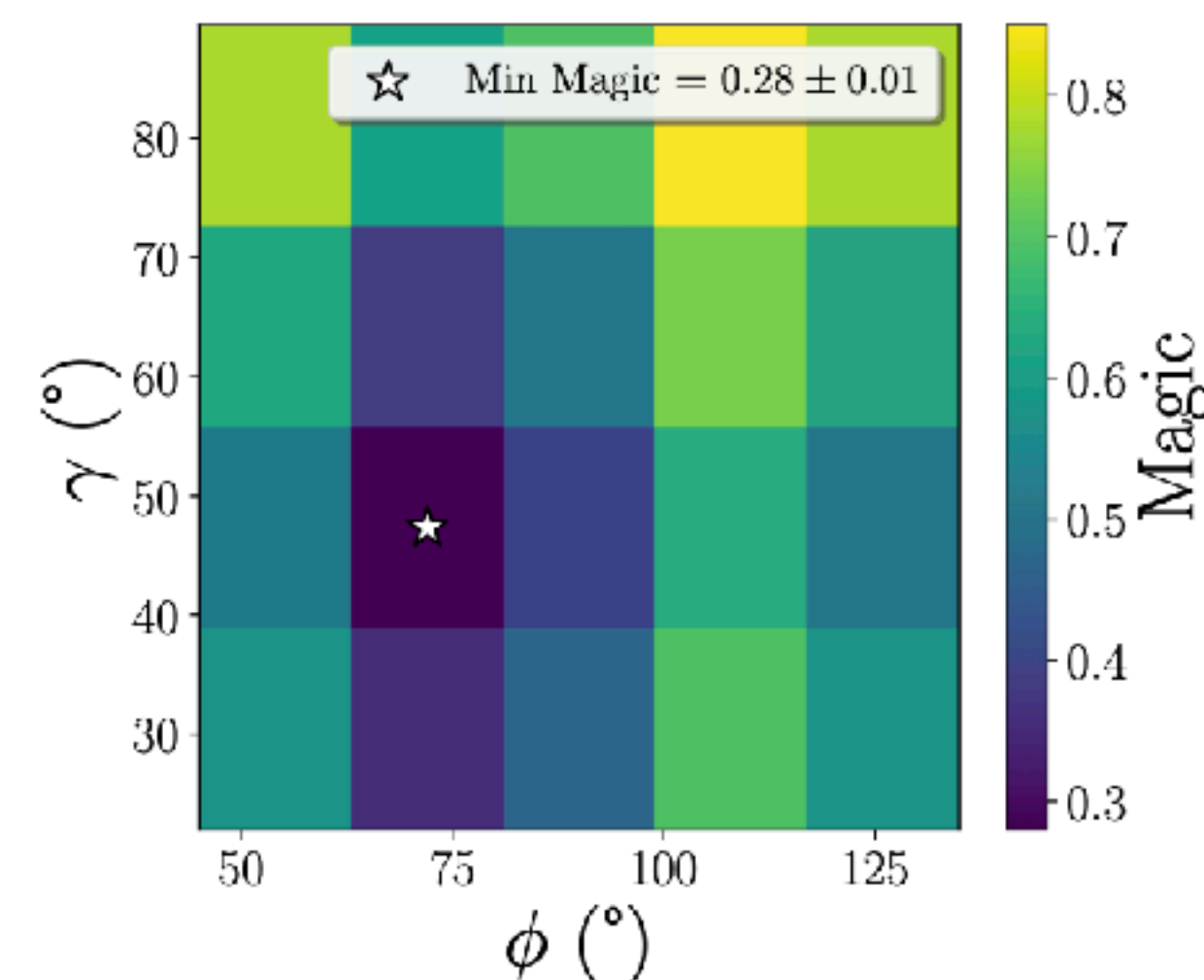


$$U_A = R_z(\alpha)R_y(\beta)R_z(\gamma), \quad U_B = R_z(\delta)R_y(\eta)R_z(\phi),$$

$$\alpha = \beta = \gamma = \delta = \eta = 0, \phi = 67.61^\circ$$

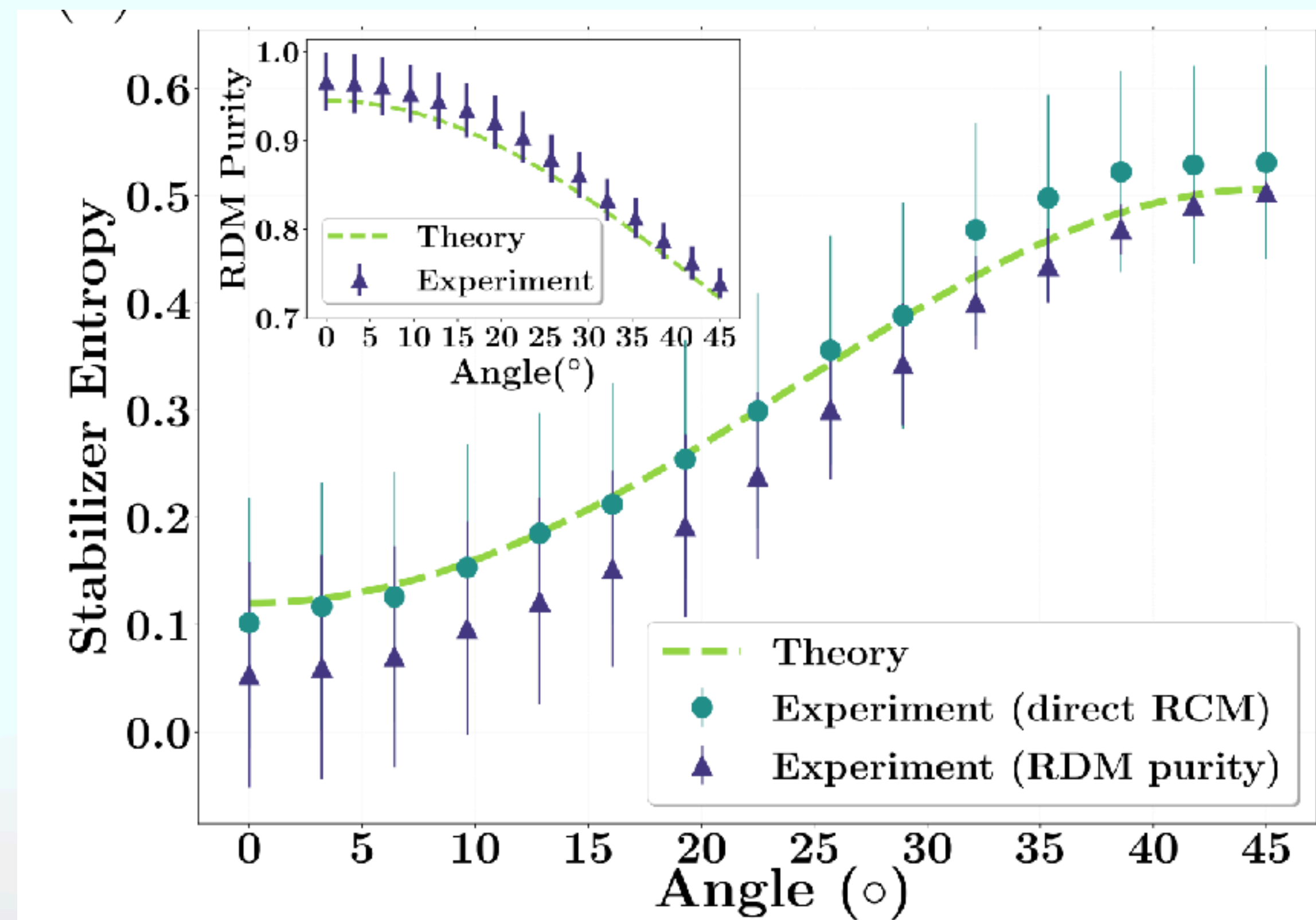
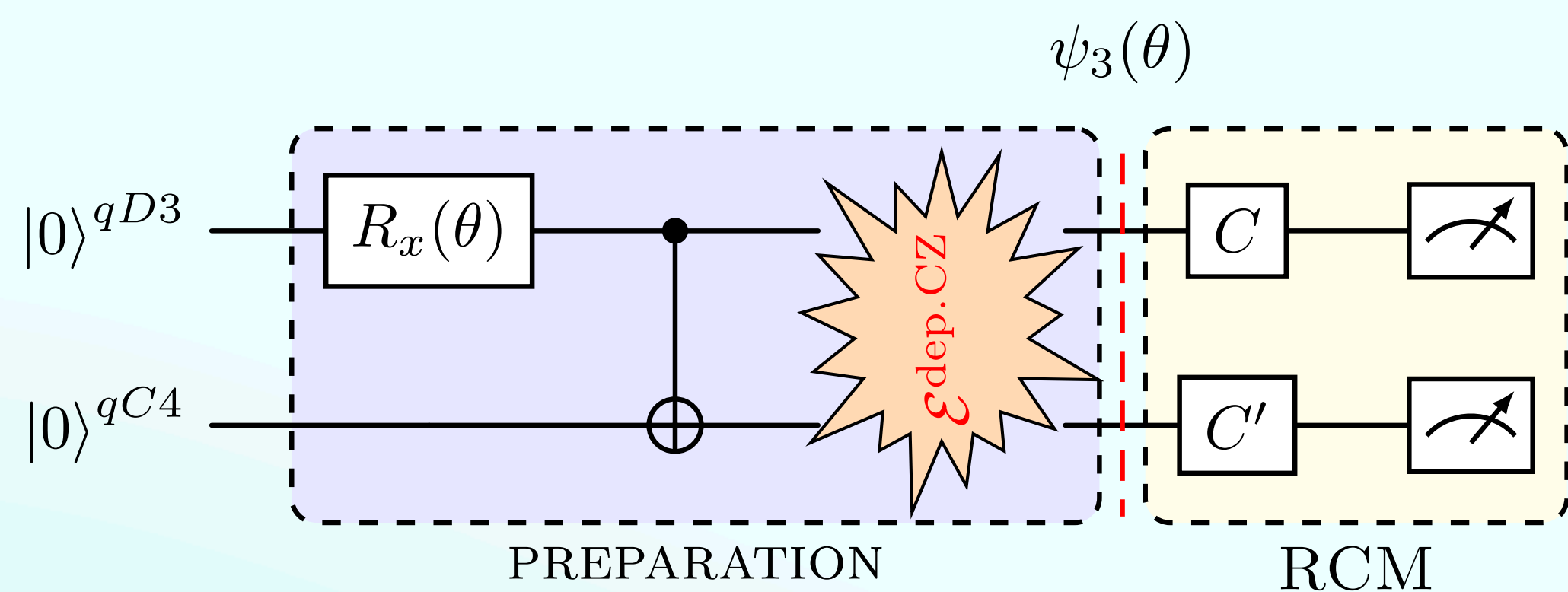
Circuit	State	Th. Purity	Exp. Purity	Th. Magic	Exp. Magic	NL-magic (from RDM purity)
Preparation	ψ_2	0.95	0.94 ± 0.04	0.44	0.42 ± 0.09	
Prep.+Erasure	ψ'_2	0.95	1.00 ± 0.05	0.3	0.3 ± 0.1	0.18 ± 0.07

Table 3: Local + Non-Local (M) magic quantum state results. Comparison of Purity and Magic for states ψ_2 (before) and ψ'_2 (after) the erasure protocol, as well as the value of non-local magic obtained from subsystem purity. The theoretical predictions for purity and magic are calculated by considering the CZ depolarizing error and compared with the experimental results after readout error mitigation. Details on the errors are reported in *Methods*.



Ono što ostane je nelokalna magija!

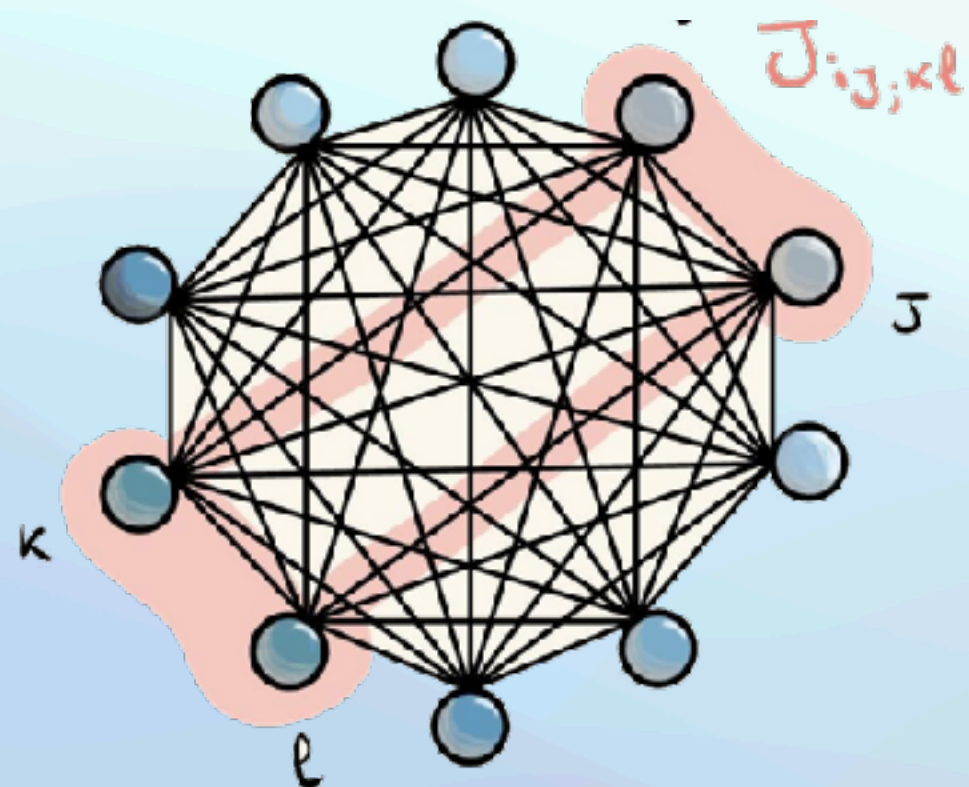
Klasa 3: Nelokalna magija (NLM)



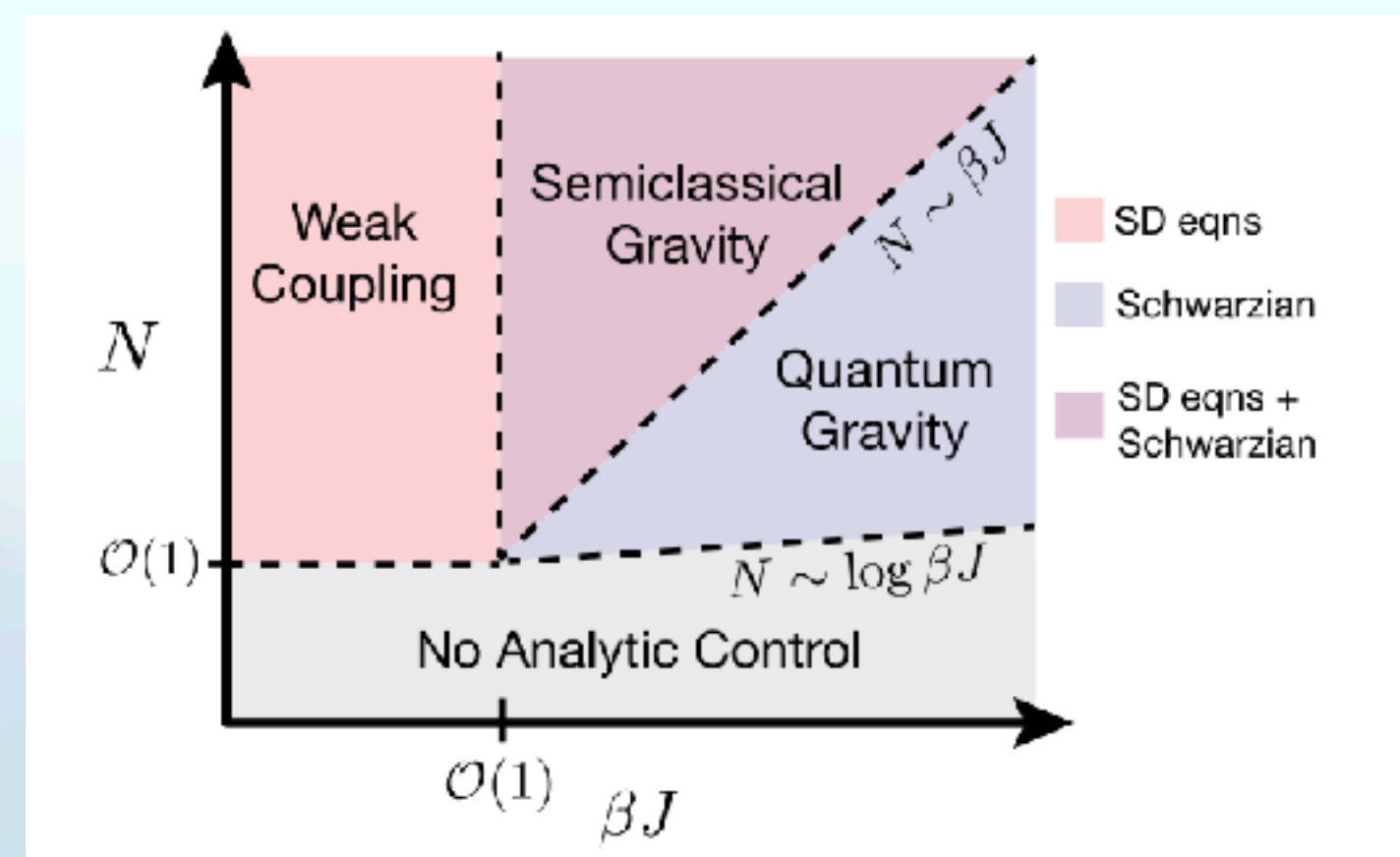
Sveukupno \rightarrow **smoking gun** eksperimentalne nelokalne magije

Perspektiva i budući rad

- Neki aspekti nelokalne magije imaju interpretaciju u AdS/CFT dualnosti? (Gravitational back-reaction is magical, arXiv:2403.07056)
- Simulacija strogo korelisanih sistema poput Sachdev-Ye-Kitaev (SYK) modela na kvantnim računarima. SYK kvantno haotični model koji opisuje neke aspekte crnih rupa.
- SYK model ima slično ponašanje niskodimenzionim teorijama gravitacije (Jackiw-Teitelboim (JT) theory i nearly-AdS2 spaces)



SYK model



Solvable limits of the SYK-4 model - PRL **126**, 030602 (2021)

Stabilizer entropy and entanglement complexity in the Sachdev-Ye-Kitaev model

[Barbara Jasser](#) ^{1,2,*}, [Jovan Odavić](#) ^{2,3,†}, and [Alioscia Hamma](#) ^{1,2,3,‡}

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Phys. Rev. B **112**, 174204 – Published 14 November, 2025

DOI: <https://doi.org/10.1103/rz86-47h3>

Novi rad na ovu temu!

Kvantna nadmoć danas?

Article

Observation of constructive interference at the edge of quantum ergodicity

<https://doi.org/10.1038/s41586-025-09526-6>

Google Quantum AI and Collaborators*

Received: 3 November 2024

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The dynamics of quantum many-body systems is characterized by quantum observables that are reconstructed from correlation functions at separate points in space and time^{1–3}. In dynamics with fast entanglement generation, however, quantum observables generally become insensitive to the details of the underlying dynamics at long times due to the effects of scrambling. To circumvent this limitation and enable access to relevant dynamics in experimental systems, repeated time-reversal protocols have been successfully implemented⁴. Here we experimentally measure the second-order out-of-time-order correlators (OTOC⁽²⁾)^{5–28} on a superconducting quantum

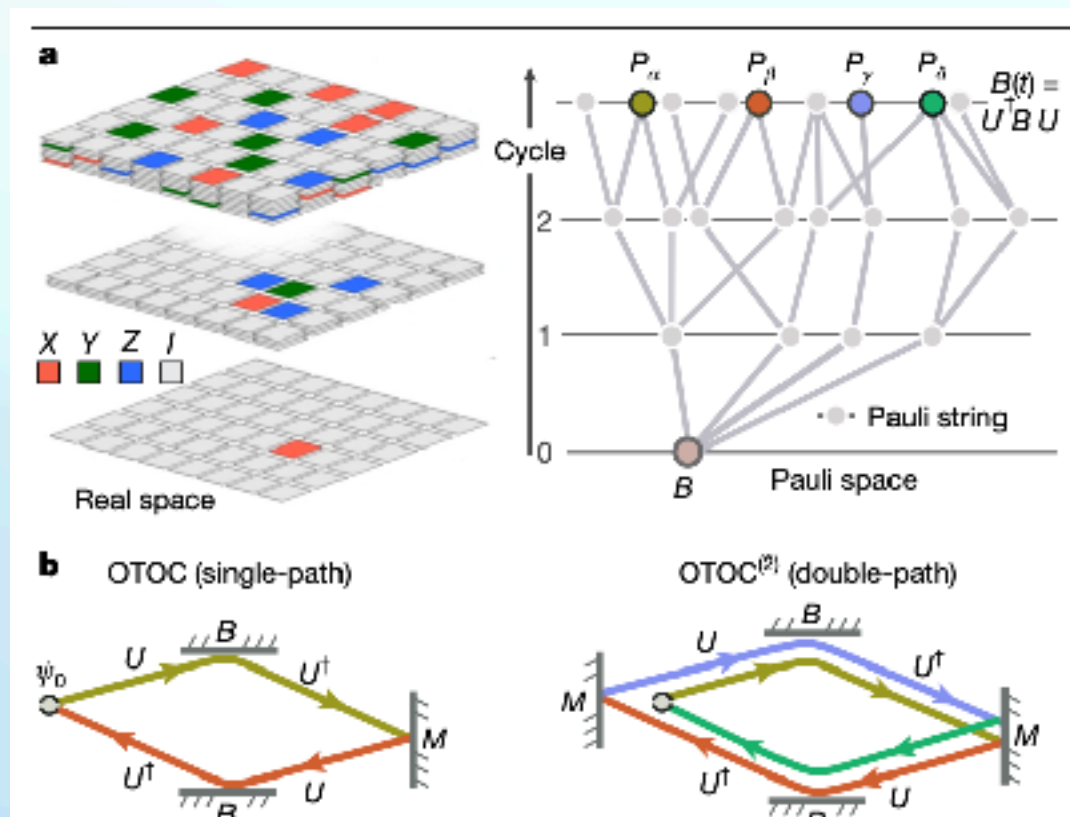
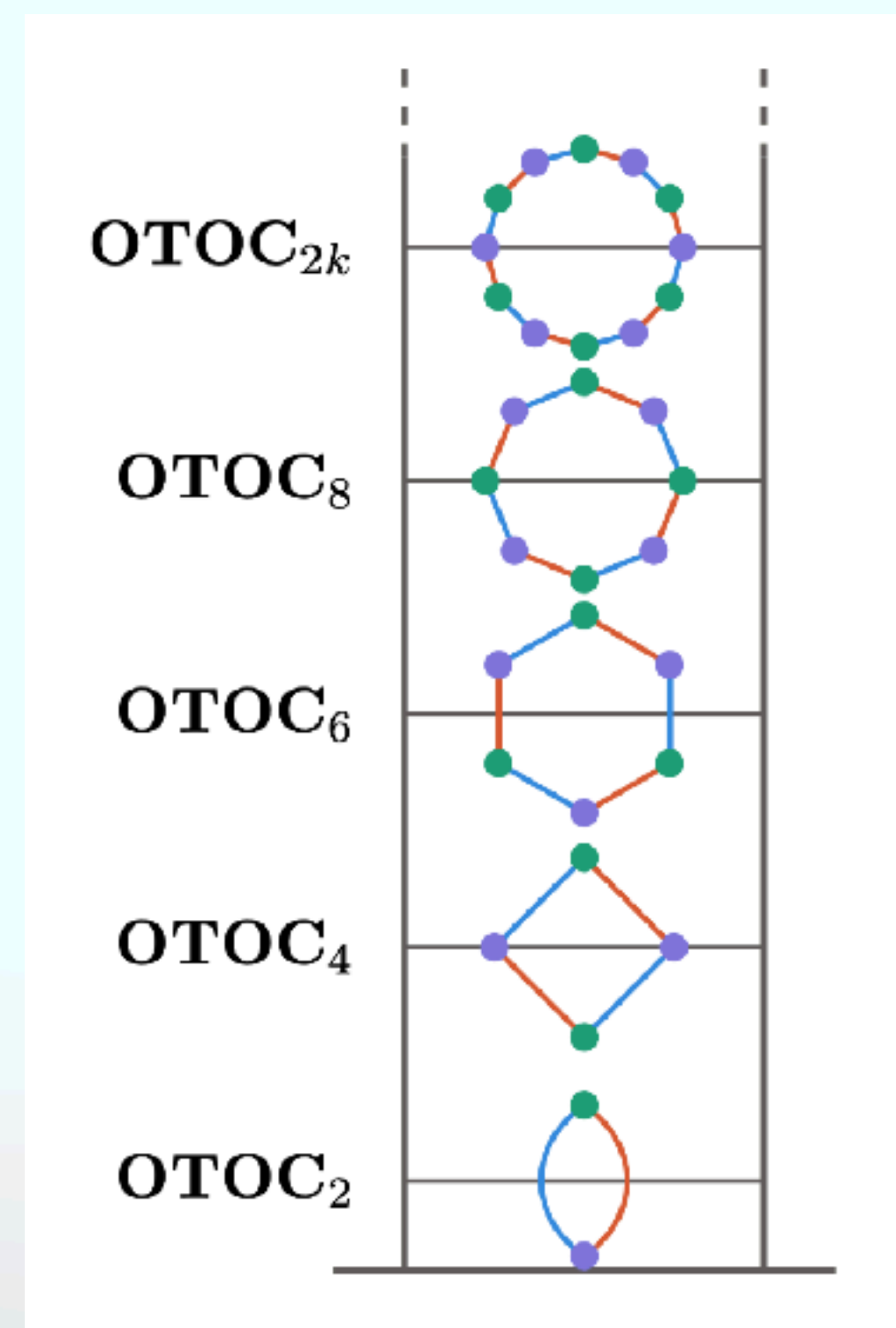


Fig. 1 | OTOCs as interferometers. **a**, When dynamical protocols involve echoing, the Heisenberg picture of the operator evolution is the natural framework for studying dynamics. **b**, OTOC and OTOC⁽²⁾ can be viewed as time interferometers, which highlights their capability of refocusing on desired details and echoing out unwanted dynamics. See text for the definition of parameters.



Jakovljeve lestve kompleksnosti



Hvala na pažnji!

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