Quantum Information & Black Holes: the Sachdev-Ye-Kitaev (SYK) Model

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Abstract

The Sachdev-Ye-Kitaev (SYK) model plays a crucial role in understanding both the physics of strange metals and the microscopic theory of two-dimensional quantum gravity [1, 2]. This strongly interacting quantum many-body system, characterized by random all-to-all couplings, is analytically tractable and is famous for its connection to black hole physics [3]. Recently, we investigated complexity transitions in an interpolated (mass-deformed) variant of the SYK model, incorporating both four-body (SYK-4) and two-body (SYK-2) interactions. Using various quantum information tools that include entanglement measures, the universality of entanglement spectrum statistics, entanglement spectrum anti-flatness, and non-stabilizerness we characterize the distinctions between ground states and highly excited states within the model's spectrum [4].

The background

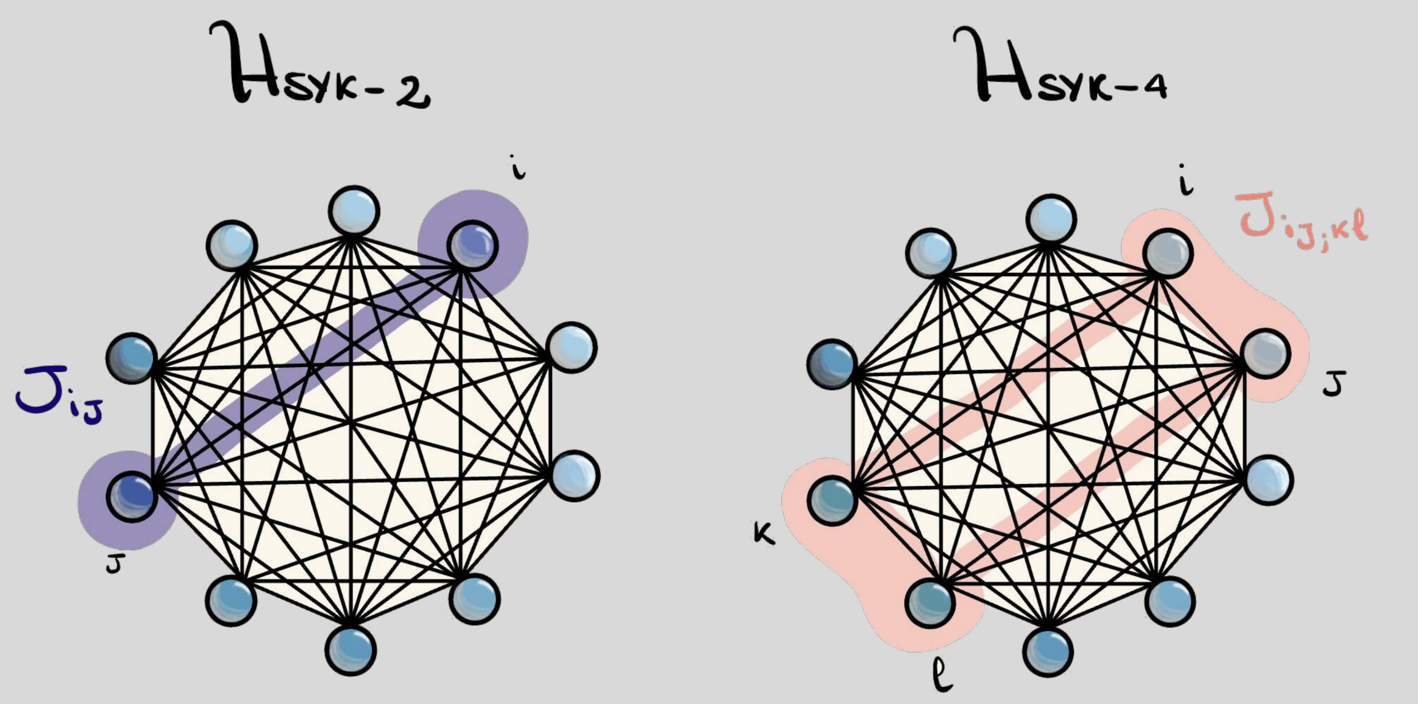
The generalized Sachdev-Ye-Kitaev model describes a q-body all-to-all interaction between N Majorana fermionic modes. The Hamiltonian reads

$$H_{\mathrm{SYK}-q} = (i)^{q/2} \sum_{1\le i_1<\ldots<i_q\le N} J_{i_1 i_2 \ldots i_q} \chi_{i_1}\chi_{i_2}\cdots\chi_{i_q} \tag{1}$$

with q being an even integer number and χ_i are Majorana operators. In the SYK model the couplings $J_{i_1,i_2,\ldots,i_q}$ are identical, independent distributed $(\mathbf{i.i.d.})$ Gaussian variables with vanishing mean and variance

$$\overline{J_{i_1,i_2,\ldots,i_q}} = 0\ ; \quad \overline{J^2_{i_1,i_2,\ldots,i_q}} = \frac{(q-1)!J}{N^{q-1}}\ , \tag{2}$$

making the system disordered. The model's low-energy physics reproduces some of the dynamical aspects of black holes! We can visualize the all-to-all interactions as a complete graph where the sites are the nodes and couplings are the lines. See below ↓



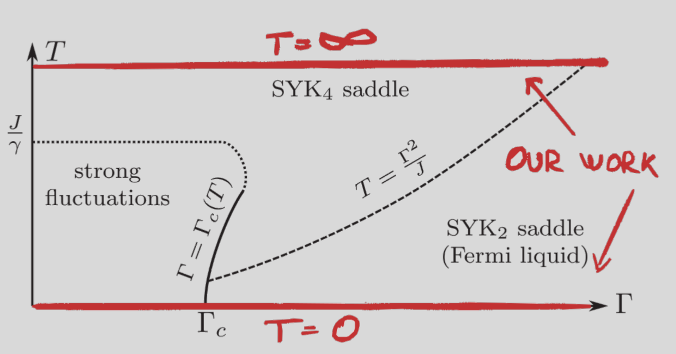
We consider the following variants of the model:

- $q = 2$, the model constitutes a random free fermion model
- $q \ge 4$, the model is considered strongly interacting and chaotic and surprisingly *analytically tractable* in the large N limit

The interpolated (mass-deformed) model [5] is defined as

$$H_g := (1-g)H_{\mathrm{SYK}-4} + gH_{\mathrm{SYK}-2}, \quad g\in[0,1]. \tag{3}$$

There exists a transition in quantum complexity depending on interpolation parameters between the models [6], and below we show the phase diagram (mind the notation $g \to \Gamma$)



A. V. Lunkin, A. Yu. Kitaev, and M. V. Feigel'man PRL **125**, 196602 (2020)

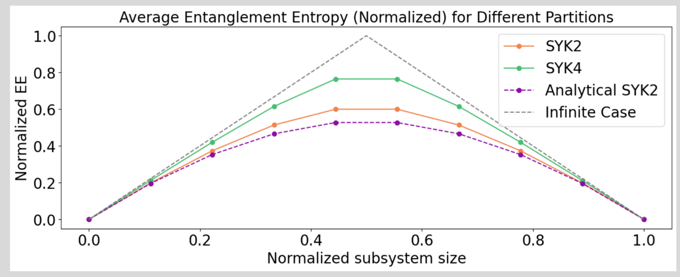
- Using field-theory techniques and the mean-field approximation, the authors extract the Majorana Green's function from which they infer the phase diagram
  - We focus on the particular parts of the diagram and take a *in silico* approach! [4]

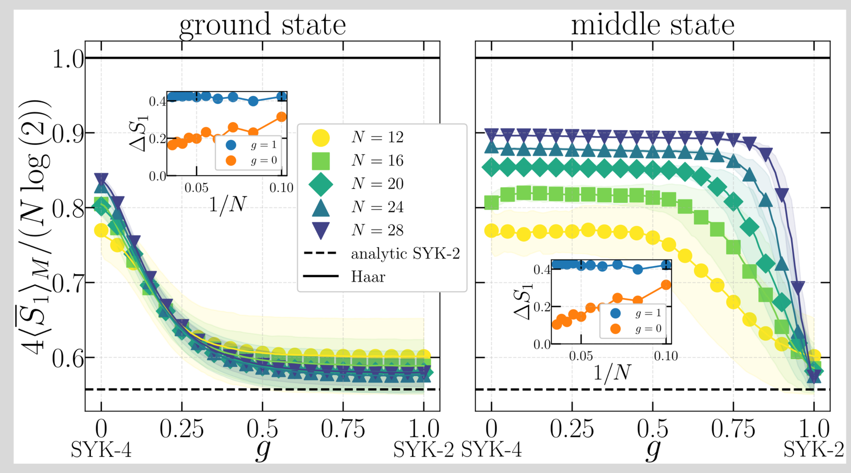
Our approach - arXiv:2502.03093 [4]

In our work [4], we lay the groundwork for exploring key questions regarding the complexity of SYK model eigenstates from the many-body and quantum information theory perspective. Both disciplines are well-founded and offer robust toolsets that enable the exploration of different aspects of quantum complexity.

Via exact diagonalization (*in silico* approach), we obtain the energy spectrum of H_g together with the *ground state* and *middle spectrum state* for many realizations of the disorder. We focus on the following quantities:

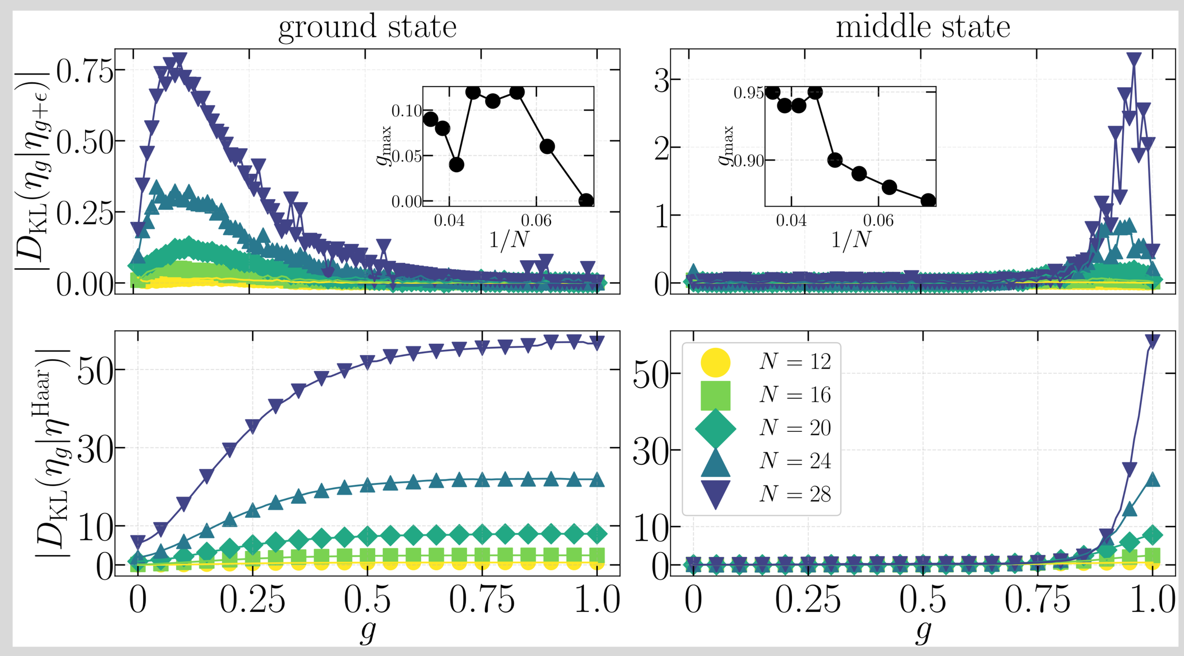
- Entanglement Entropy $S_1 = -\mathrm{Tr}[\rho_R \log(\rho_R)]$





EE between the model variants together with the Page curve and the analytical results for the SYK-2 model. Volume law EE.

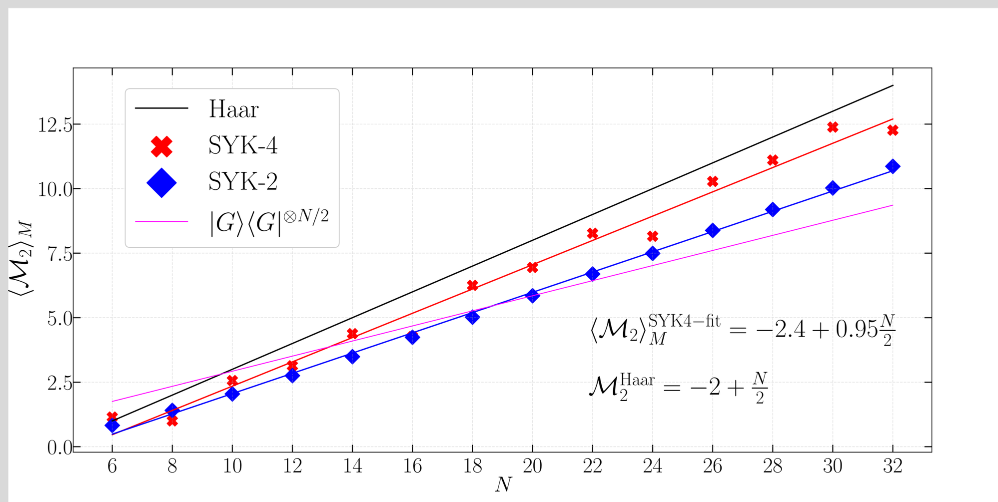
- Reduced Density Matrix $\rho_R = \mathrm{Tr}_{\bar{R}}|\Psi\rangle\langle\Psi|$ eigenvalues $\{p_i\}_{i=1}^{2^{|R|}}$
  - Utilize the Kullback-Leibler (KL) divergence $D_{KL}(p||q) := \sum_i p_i(\log p_i - \log q_i)$ with the reference maximally chaotic Haar value $\eta^{\mathrm{Haar}}(x)$
  - introduced the KL fidelity $|D_{\mathrm{KL}}(\eta_g|\eta_{g+\epsilon})|$



Renormalized RDM eigenvalues of half-subsystem.
Ground state SYK-2 phase dominates, while in the middle states SYK-4 dominates.

- Stabilizer Entropy

$$\mathcal{M}_\alpha(\Psi) = \frac{1}{1-\alpha}\log_2\left(2^{-N}\sum_{P\in\mathrm{Pauli}}|\mathrm{Tr}(|\psi\rangle\langle\psi|P)|^{2\alpha}\right). \tag{4}$$



Non-stabilizerness or magic (resource for quantum advantage).

Conclusions and Outlook

- Discovering the rich phase diagram from the analysis of the eigenstates using the quantum info toolbox; non-local magic as key resource; deviations from maximally chaotic (Haar) states
- Large N limit non-stabilizers of the SYK model; sparsification model and the properties of the eigenstates of the generalized SYK model to simulate traversable wormholes on a quantum computer

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